

Inelastic Diffraction and Color-Singlet Gluon Clusters in High-Energy Hadron-Hadron and Lepton-Hadron Collisions

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It is proposed that the “colorless” objects which manifest themselves in large-rapidity-gap events are color-singlet gluon clusters due to self-organized criticality (SOC), and that optical-geometrical concepts and methods are useful in examining the space-time properties of such objects. A simple analytical expression for the t dependence of the inelastic single diffractive cross section $d\sigma/dt$ (t is the four-momentum transfer squared) is derived. Comparison with existing data and predictions for future experiments are presented. The main differences and similarities between the SOC approach and the “partons in the Pomeron (Pomeron and Reggeon)” approach are discussed. [S0031-9007(99)08670-6]

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Diffraction in optics can be used as an instrument to determine the unknown wavelengths of incident waves from the known geometrical structures of scatterers and vice versa. This was pointed out and demonstrated by von Laue and his collaborators in their celebrated paper [1] eighty-five years ago. Based on this idea, a series of experiments [2] were performed in the 1950s and 1960s to measure the sizes of various nuclei by using hadron beams (as hadronic waves), where particle accelerators have been used as “super microscopes” [3]. Theoretically, the idea of using optical and/or geometrical analogies to describe high-energy hadron-nucleus and hadron-hadron collisions at small scattering angles has been discussed by many authors [4] many years ago. It is shown [4] in particular that this approach is very useful in describing hadron-hadron elastic scattering.

Since the recent observation [5] of large-rapidity-gap (LRG) events in deep-inelastic electron-proton scattering in the small- x_B region ($x_B < 10^{-2}$, say)—a kinematical region where soft gluons dominate [6], much attention [7] has been attracted by inelastic diffractive scattering processes in lepton- and hadron-induced high-energy collisions [5,7–11]. The vast interest in these kinds of events is due mainly to the fact that the occurrence of LRG’s strongly suggests the existence of certain “colorless” object(s), the “exchange” of which plays a dominating role in such processes. The colorless object which carries the quantum numbers of vacuum has been given [5,7–13] different names (Pomeron, Reggeon, etc.), and it has been suggested [12,13] that it (they) should be hadronlike and thus should have hadronlike fluxes and hadronlike structure functions. This class of inelastic processes are often called [5,7–13] “diffractive scattering processes,” because the same kind of colorless object also plays a significant role in elastic hadron-hadron scattering, and the differential cross-section data of the latter exhibit diffraction patterns similar to those observed in optics.

What *are* such colorless objects? Can their occurrence, their properties, and their effects be understood in terms of QCD? What is the relationship between such colorless

objects and inelastic diffraction in optics? Can optical-geometrical concepts and methods be used to describe, say, inelastic diffractive scattering processes at large values of invariant momentum transfer, $|t| \geq 0.2 \text{ GeV}^2$? In order to answer these questions, it seems useful to recall and/or note the following.

(A) A number of experimental facts [6–9] and theoretical arguments [14] suggest that the colorless objects which the beam particles encounter in LRG events are color-singlet gluon clusters.

(B) The characteristic properties of the gluons—especially the local gluon-gluon coupling prescribed by the QCD Lagrangian, the confinement, and the nonconservation of gluon numbers—strongly suggest that systems of interacting soft gluons are *open, dynamical, complex* systems with *many degrees of freedom*, and that such systems are in general *far from equilibrium*. This means in particular that, since soft gluons can be emitted and/or absorbed at any time, and everywhere in such a system, it is neither meaningful nor possible to specify the number of gluons or the amount of energy associated with the system.

(C) It has been pointed out by Bak, Tang, and Wiesenfeld (BTW) [15] that a wide class of open complex systems evolve into self-organized critical (SOC) states, and local perturbations of such critical states may propagate like avalanches caused by domino effects over all length scales. Such a long-range correlation effect eventually terminates after a total time T , having reached a final amount of dissipative energy, and having effected a total spatial extension S . The quantity S is called by BTW [15] the “size,” and the quantity T the “lifetime” of the “avalanche” and/or the “clusters.” It is observed [15,16] that there are many such open dynamical complex systems in the macroscopic world, and that the distributions D_S of S and the distribution D_T of T of such BTW avalanches/clusters obey power laws: $D_S(S) \propto S^{-\mu}$ and $D_T(T) \propto T^{-\nu}$, where μ and ν are positive real constants. Such characteristic behaviors are known [15,16] as the “fingerprints” of SOC. Having this and the characteristic features of the gluons mentioned in

(B) in mind, we are naturally led to the following question: Can SOC and thus BTW avalanches also exist in microscopic systems—at the level of quarks and gluons? Can SOC be the dynamical origin of color-singlet gluon clusters which play the dominating role in inelastic diffractive scattering processes?

(D) In order to answer these questions we performed a systematic analysis [17] of the data [7,8] for diffractive deep-inelastic scattering (DIS), where we made use of the following: (i) For a color-singlet gluon cluster c_0^* to be a BTW avalanche, its spatial size S has to be directly proportional to the dissipative energy, and the latter is proportional to x_P which is the energy fraction carried by c_0^* . Hence, $D_S(S)$ is expected to be related to the x_P distribution of the c_0^* 's in LRG events of DIS. (ii) In such events, x_P and the “diffractive structure function” $F_2^{D(3)}$ are measurable quantities. This, together with the observation that the impulse approximation is applicable to c_0^* 's (since the interactions between the struck c_0^* and any other neighboring color singlets is expected to be similar to those between hadrons, the former can also be considered as Van der Waal's type of interactions), implies that $D_S(S)$ can be extracted in a similar manner as the x_B distribution of valence quarks in normal events. Furthermore, if the c_0^* 's are indeed due to SOC, there should be c_0^* 's of all lifetimes (T 's). This means, for a given interaction time τ_{int} , there are always c_0^* 's whose T 's are longer than τ_{int} . (iii) The interaction time τ_{int} can be estimated by making use of the uncertainty principle; to be more precise [cf. Eq. (10) of Ref. [17]], for fixed $|\vec{P}|$ and Q^2 values, $\tau_{\text{int}} \propto x_B$ for $x_B \ll 1$. Hence, in the small- x_B region, information about the lifetime distribution can be obtained by examining the x_B dependence of $F_2^{D(3)}$. The results of the analysis can be summarized as follows: The size and the lifetime distributions are indeed of the form $D_S(S) \propto S^{-\mu}$ and $D_T(T) \propto T^{-\nu}$, where $\mu \approx \nu \approx 2$. Furthermore, for obvious reasons, $D_S(S)$ and $D_T(T)$ are expected to retain their power-law behaviors with exactly the same exponent in all Lorenz frames. Comparisons between these results and those obtained for inelastic diffractive γP [9], pP [10], and $\bar{p}P$ [11] processes have also been made [17], and the following picture emerges: In an inelastic diffractive scattering process off proton the beam particle encounters one of the color-singlet gluon clusters which in general exist partly inside and partly outside the confinement region of the proton. The size and the lifetime distributions of the clusters exhibit universal power-law behaviors, which imply in particular that such gluon clusters are *not* hadronlike in the sense that they have *neither* a typical size *nor* a typical lifetime. Furthermore, the fact [17] that the data [8] *cannot* accommodate the simple factorization assumption [12], in which a universal Pomeron flux with a unique hadronlike Pomeron structure function exists, gives further support to the proposed SOC picture because a BTW cluster *cannot* have a universal static structure. With these characteristic properties of the colorless objects in mind, we see [13] an overlap

between the SOC picture and the partonic picture for Pomeron and/or Pomeron and Reggeon [12,13] in which, beside the Pomeron, exchange of (in general an infinite number of) subleading Regge trajectories are possible. In fact, it has been reported [8] that very good agreement can be achieved between the data [8] and these types of models. Hence, in order to differentiate between the two approaches, it is not only useful but also necessary to examine the corresponding predictions for the dependence on the invariant momentum transfer t .

(E) In optics, Fraunhofer diffraction can be observed when the wavelength of the light is less than the linear dimension of the scatterer, and when the light source and the detecting screen are far from the scatterer. The incident light rays are considered as plane waves (wave vector \vec{k} , frequency $\omega \equiv |\vec{k}|$). After the scattering, the field originating from the scatterer can be written as the product of $R^{-1} \exp(i|\vec{k}'|R)$ and the scattering amplitude $f(\vec{k}, \vec{k}')$. Here, \vec{k}' is the wave vector of the scattered light in the direction of observation, $\omega' \equiv |\vec{k}'|$ is the corresponding frequency, and R is the distance between the scatterer and the observation point. By choosing a coordinate system in which the z axis coincides with \vec{k} , $f(\vec{k}, \vec{k}')$ can be expressed [18] as

$$f(\vec{q}) = (2\pi)^{-2} \int \int_{\Sigma} d^2\vec{b} \alpha(\vec{b}) e^{-i\vec{q}\cdot\vec{b}}. \quad (1)$$

Here, $\vec{q} \equiv \vec{k}' - \vec{k}$ determines the change in the wave vector due to diffraction, \vec{b} is the impact parameter, $\alpha(\vec{b})$ is the corresponding amplitude in the two-dimensional b space (here, the xy plane), and the integration extends over the region Σ in which $\alpha(\vec{b})$ is different from zero. It is well known that elastic scattering at small angles can be deduced [18] from this equation under the additional condition $|\vec{k}'| = |\vec{k}| = \omega' = \omega$, where geometry dictates that \vec{q} is approximately perpendicular to \vec{k} and to \vec{k}' . In such cases, $\vec{q} \approx \vec{q}_{\perp}$ where \vec{q}_{\perp} represents its projection on the xy plane of the chosen coordinate system. We note the general case, in which $\vec{k}' \neq \vec{k}$ and $\omega' \neq \omega$, corresponds to inelastic scattering, for which we can write

$$f_{\text{inel}}(\vec{q}_{\perp}) = (2\pi)^{-2} \int \int_{\Sigma} d^2\vec{b} \alpha_{\text{inel}}(\vec{b}) e^{-i\vec{q}_{\perp}\cdot\vec{b}}. \quad (2)$$

Here both $\alpha_{\text{inel}}(\vec{b})$ and $f_{\text{inel}}(\vec{q}_{\perp})$ in general depend on the transfer of energy and/or longitudinal momentum. Furthermore, if the scatterer is symmetric with respect to the scattering axis, Eq. (2) can also be expressed by using an integral representation for J_0 , as

$$f_{\text{inel}}(q_{\perp}) = (2\pi)^{-1} \int_0^{\infty} b db \alpha_{\text{inel}}(b) J_0(q_{\perp} b), \quad (3)$$

where q_{\perp} and b stand for $|\vec{q}_{\perp}|$ and $|\vec{b}|$, respectively.

Keeping the facts mentioned in (A)–(E) in mind, we now examine the scatterer quantitatively in the rest frame of the proton target. We choose a right-handed Cartesian coordinate system with its origin O at the center of the proton and the z axis in the direction of the incident beam

particle which is considered pointlike as it goes through the xy plane at point $B \equiv (0, b)$. That is, the incident beam and the center O of the proton determine the scattering plane (yz plane) of the collision event, where the distance \overline{OB} is the corresponding impact parameter b . Since we are dealing with inelastic scattering (where the momentum transfer also in the longitudinal direction can be large), it is possible to envisage that the scattering takes place at one point in space, and effectively [19] at the point B , where it meets colorless gluon clusters. The latter are BTW avalanches due to SOC initiated by local interactions in systems of soft gluons. Since gluons carry color, the interactions which lead to the formation of gluon clusters must take place inside the confinement region of the proton. This means, while a considerable part of the created color-singlet clusters can be outside the proton, the location A where such an avalanche is initiated must be *inside* the proton. That is, in terms of $\overline{OA} \equiv r$, $\overline{AB} \equiv R_A(b)$, and proton's radius r_p , we have $r \leq r_p$ and $[R_A(b)]^2 = b^2 + r^2 - 2br \cos(\angle BOA)$. For a given impact parameter b , the relevant quantity is the mean distance $\langle R_A^2(b) \rangle^{1/2} = (b^2 + a^2)^{1/2}$, $a^2 \equiv \frac{3}{5}r_p^2$, which is obtained by averaging over all allowed locations of A in the confinement region. That is, we can model *the effect of confinement* in avalanche formation by picturing that all such BTW avalanches, in particular those which contribute to scattering events characterized by a given b , are initiated from an "effective initial point" $\langle A_b \rangle$. Note also that, since an avalanche is a dynamical object, it in general expands and propagates within its lifetime (in any one of the 4π directions away from $\langle A_b \rangle$). It is envisaged that the expansion and propagation of such an avalanche cannot proceed without limits. This is not only because the gluon density is expected to decrease with increasing distance from O but also because color forces between the constituents of c_0^* increase (consistent with confining potentials) when their distance becomes larger. That is, those constituents "going too far" will be "pulled back" to the proton's confinement region.

An explicit expression for the amplitude $\alpha_{\text{inel}}(b)$ in Eq. (3) can be readily written down by taking the following into account: (α) SOC dictates that there are avalanches of all sizes (S_i 's) and that the number density of avalanches of S_i is $D_S(S_i) \propto S_i^{-\mu}$, where experiments show [17] $\mu \approx 2$. (β) Since the interaction between the struck c_0^* and an other color singlets should be of Van der Waals type, it can simply be "carried away" by the beam particle. Simple geometrical considerations suggest that the chance for an avalanche of size (i.e., volume) S_i to be hit (on the plane perpendicular to the incident axis) should be proportional to $S_i^{2/3}$. (γ) Since, for a given b , the distance in space between $\langle A_b \rangle$ and $B \equiv (0, b)$ is $(b^2 + a^2)^{1/2}$, the number of avalanches which pass a unit area on the shell of radius $(b^2 + a^2)^{1/2}$ centered at $\langle A_b \rangle$ is proportional to $(b^2 + a^2)^{-1}$, provided that (due to causality) the lifetimes (T 's) of these avalanches are not shorter than $\tau_{\text{min}}(b)$. The latter is the time interval for

an avalanche to travel (with constant velocity, say) from $\langle A_b \rangle$ to B , that is, $\tau_{\text{min}}(b) \propto (b^2 + a^2)^{1/2}$, and only those avalanches having lifetimes $T \geq \tau_{\text{min}}(b)$ can contribute to such an event. Now, since avalanches are due to SOC and the chance for an avalanche of lifetime T to exist is $D_T(T) \propto T^{-\nu}$, where [17] $\nu \approx 2$, the proper fraction can be obtained by integrating T^{-2} over T from $\tau_{\text{min}}(b)$ to infinity, which is $(b^2 + a^2)^{-1/2}$.

Hence, for inelastic diffractive scattering processes in which the beam particles encounter avalanches of size S_i , the amplitude which we denote by $\alpha_{\text{inel}}(b | S_i)$ can be obtained from the square root of $D_S(S_i)$ mentioned in (α), and by taking the weighting factors mentioned in (β) and (γ) into account. The result is

$$\alpha_{\text{inel}}(b | S_i) = \text{const} \times S_i^{-1/3} (b^2 + a^2)^{-3/2}. \quad (4)$$

By inserting this into Eq. (3), we obtain the corresponding amplitude in \vec{q}_\perp space: $f_{\text{inel}}(q_\perp | S_i)$. Here, $q_\perp = |\vec{q}_\perp| \approx \sqrt{|t|}$ is the corresponding momentum transfer. The integration can be carried out analytically [21], and the result (after the summation over i) for the inclusive inelastic single diffractive differential cross section $d\sigma/dt$ is

$$d\sigma/dt = C \exp(-2a\sqrt{|t|}). \quad (5)$$

Here $a^2 \equiv \frac{3}{5}r_p^2$, and C is an unknown constant. Note that the value for a is *the same* for different incident energies, and for different projectiles: γ^* , γ , p , \bar{p} , etc.

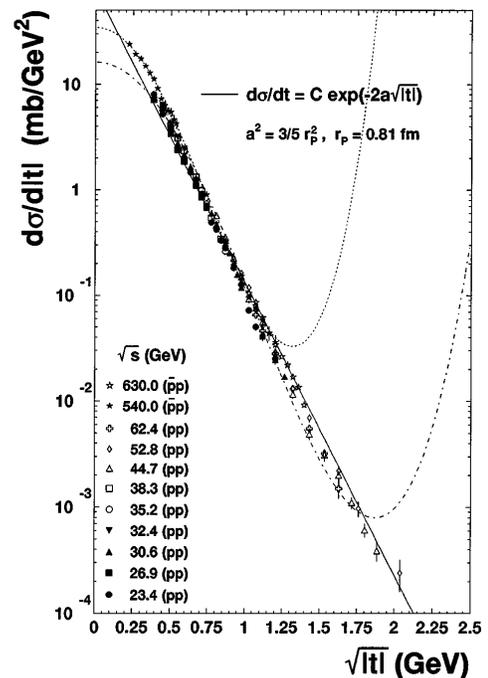


FIG. 1. The $d\sigma/dt$ data taken from Refs. [10,11] are plotted against $\sqrt{|t|}$ in the measured kinematical range. The solid line is our result. The dotted and dot-dashed lines show the conventional fits $d\sigma/dt \propto \exp(Bt + Ct^2)$. The former is the UA4 fit to their data [11] with $B = 8.0 \pm 0.1 \text{ GeV}^{-2}$ and $C = 2.3 \pm 0.1 \text{ GeV}^{-4}$. The latter is a fit to the same expression for all of the data points in this figure, where $B = 5.7 \pm 0.1 \text{ GeV}^{-2}$ and $C = 0.8 \pm 0.1 \text{ GeV}^{-4}$.

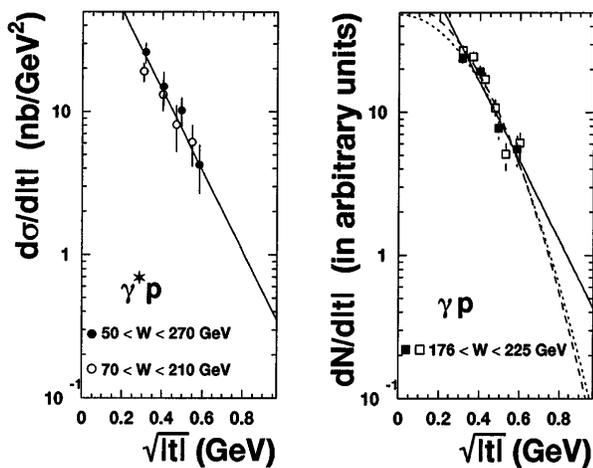


FIG. 2. The $d\sigma/d|t|$ data from Refs. [7,9] for γ^* and γ -induced reactions (circles and squares, respectively) are plotted against $\sqrt{|t|}$. Here, the empty circles and squares are the data from (the transparencies of) Ref. [7] while the solid ones are those from Ref. [9]. The solid lines represent our result. The dashed and dotted lines are fits given in Refs. [7,9]: $d\sigma/dt \propto \exp(-b|t|)$, where $b = 7.3 \pm 0.9 \pm 1.0 \text{ GeV}^{-2}$ and $b = 6.8 \pm 0.9 + 1/2|t| - 1.1 \text{ GeV}^{-2}$.

To compare this result with experiments, we calculate a by using its definition $a^2 \equiv \frac{3}{5}r_p^2$ and the experimental value [22] for r_p , which is 0.81 fm. We note, while the “slope” $2a$ depends only on the radius of the target hadron, the normalization constant C is expected to be different for different projectiles. The comparisons are shown in Figs. 1 and 2.

A few remarks should be made in this connection: (i) The $d\sigma/dt$ data for pp , $\bar{p}p$, γp , and γ^*p reactions are consistent with *no* energy (\sqrt{s} and/or W) dependence. In particular, in contrast to elastic pp scattering, there is no indication of “shrinkage.” (ii) The figures show the following: In a limited range of $\sqrt{|t|}$, curves with very different t behaviors can yield rather similar results. (iii) The existing data show that the coefficients of $\sqrt{|t|}$, t , or t^2 in the curves for $d\sigma/dt$, are consistent with no M_x dependence. This is, in particular, the case for γp reactions (see, in this connection, the second paper of Ref. [9]). (iv) A simple analytic expression for $d^2\sigma/dt dx_p$ can be readily obtained by taking $S_i \propto x_p$ into account in Eq. (4) and by integrating over b . The result, which is shown to be in good agreement with the existing data for pp and $\bar{p}p$ reactions, will be presented in Ref. [20]. Evidently, further experiments, especially measurements of $d\sigma/dt$ and $d^2\sigma/dt d(M_x^2/W^2)$ for $\gamma p \rightarrow Xp$ at larger $|t|$ values, would be helpful.

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