

Formation of color-singlet gluon clusters and inelastic diffractive scattering

C. Boros,^{*} Meng Ta-chung,[†] R. Rittel, K. Tabelow, and Zhang Yang[‡]
Institut für Theoretische Physik, FU Berlin, 14195 Berlin, Germany

(Received 10 May 1999; published 5 April 2000)

This is the extensive followup report of a recent Letter in which the existence of self-organized criticality (SOC) in systems of interacting soft gluons is proposed, and its consequences for inelastic diffractive scattering processes are discussed. It is pointed out that color-singlet gluon clusters can be formed in hadrons as a consequence of SOC in systems of interacting soft gluons, and that the properties of such spatiotemporal complexities can be probed experimentally by examining inelastic diffractive scattering. Theoretical arguments and experimental evidence supporting the proposed picture are presented—together with the result of a systematic analysis of the existing data for inelastic diffractive scattering processes performed at different incident energies and/or by using different beam particles. It is shown in particular that the size and the lifetime distributions of such gluon clusters can be directly extracted from the data, and the obtained results exhibit universal power-law behaviors—in accordance with the expected SOC fingerprints. As further consequences of SOC in systems of interacting soft gluons, the t dependence and the (M_x^2/s) dependence of the double differential cross sections for inelastic diffractive scattering off a proton target are discussed. Here t stands for four-momentum-transfer squared, M_x for the missing mass, and \sqrt{s} for the total c.m. system energy. It is shown that the space-time properties of the color-singlet gluon clusters due to SOC, discussed above, lead to simple analytical formulas for $d^2\sigma/dt d(M_x^2/s)$ and for $d\sigma/dt$, and that the obtained results are in good agreement with the existing data. Further experiments are suggested.

PACS number(s): 13.85.Hd, 12.40.Nn, 13.60.Hb

I. INTERACTING SOFT GLUONS IN THE SMALL- x_B REGION OF DIS

A number of striking phenomena have been observed in recent deep-inelastic electron-proton scattering (DIS) experiments in the small- x_B region. In particular it is seen that the contribution of the gluons dominates [1] and that large-rapidity-gap (LRG) events exist [2–4]. The latter show that the virtual photons in such processes may encounter “colorless objects” originating from the proton.

The existence of LRG events in these and other [5,6] scattering processes has attracted much attention, and there has been much discussion [2–11] on problems associated with the origin and/or the properties of such “colorless objects.” Reactions in which the “exchange” of such “colorless objects” dominates are known in the literature [3,4,8,9] as “diffractive scattering processes.” While the concepts and methods used by different authors in describing such processes are in general very much different from one another, all the authors (experimentalists as well as theorists) seem to agree on the following [9] (see also Refs. [2–8,10–12]): (a) Interacting soft gluons play a dominating role in understanding the phenomena in the small- x_B region of DIS in general and in describing the properties of LRG events in particular. (b) Perturbative QCD should be, and can be, used to describe the LRG events associated with high-transverse-momentum (p_\perp) jets which have been observed at the DESY

ep collider HERA [10] and at the Fermilab Tevatron [7]. Such events are, however, rather rare. For the description of the bulk of LRG events, concepts and methods beyond perturbative QCD, for example, Pomeron models [8] based on Regge phenomenology, are needed. It was suggested a long time ago (see the first two papers in Ref. [8]) that, in QCD language, “Pomeron exchange” can be interpreted as the “exchange of two or more gluons” and that such results can be obtained by calculating the corresponding Feynman diagrams. It is generally felt that nonperturbative methods should be useful in understanding “small- x_B phenomena,” but the question as to whether or how perturbative QCD (pQCD) plays a role in such nonperturbative approaches does not have a unique answer.

In a recent Letter [12], we proposed that the “colorless objects” which play the dominating role in LRG events are color-singlet gluon clusters due to self-organized criticality, and that optical-geometrical concepts and methods are useful in examining the space-time properties of such objects.

The proposed picture [12] is based on the following observation: In a system of soft gluons whose interactions are not negligible, gluons can be emitted and/or absorbed at any time and everywhere in the system due to color interactions between the members of the system as well as due to color interactions of the members with gluons and/or quarks and antiquarks outside the system. In this connection it is important to keep in mind that gluons interact directly with gluons and that *the number of gluons in a system is not a conserved quantity*. Furthermore, since in systems of interacting soft gluons the “running coupling constant” is in general greater than unity, nonperturbative methods are needed to describe the local interactions associated with such systems. That is, such systems are in general extremely complicated; they are not only too complicated (at least for us) to take the details

^{*}Present address: Centre for Subatomic Structure of Matter (CSSM), University of Adelaide, Australia 5005.

[†]Email address: meng@physik.fu-berlin.de

[‡]Present address: Chinese Academy of Sciences, Institute of Theoretical Physics, POB 2735, Beijing 100080, China.

of local interactions into account (for example, by describing the reaction mechanisms in terms of Feynman diagrams), but also too complicated to apply well-known concepts and methods in conventional equilibrium statistical mechanics. In fact, the accumulated empirical facts about LRG events and the basic properties of gluons prescribed by QCD are forcing us to accept the following picture for such systems:

A system of interacting soft gluons can be, and should be, considered as *an open, dynamical, complex system with many degrees of freedom*, which is in general *far from equilibrium*.

In our search for an appropriate method to deal with such complex systems, we are led to the following questions: Do we see comparable complex systems in nature? If yes, what are the characteristic features of such systems, and what can we learn by studying such systems?

II. CHARACTERISTIC FEATURES OF OPEN DYNAMICAL COMPLEX SYSTEMS

Open, dynamical, complex systems which are in general far from equilibrium are *not* difficult to find in nature—at least *not* in the macroscopic world. Such systems have been studied, and in particular the following were observed by Bak, Tang, and Wiesenfeld (BTW) some time ago [13]: This kind of complex system may evolve into self-organized critical states which lead to fluctuations extending over all length and time scales, and that such fluctuations manifest themselves in the form of spatial and temporal power-law scaling behaviors showing properties associated with fractal structure and flicker noise, respectively.

To be more precise, BTW [13] and many other authors [14] proposed, and demonstrated by numerical simulations, the following: Open, dynamical, complex systems of locally interacting objects which are in general far from equilibrium can evolve into self-organized structures of states which are barely stable. A local perturbation of a critical state may “propagate,” in the sense that it spreads to (some) nearest neighbors, and then to the next-nearest neighbors, and so on in a “domino effect” over all length scales, the size of such an “avalanche” can be as large as the entire system. Such a “domino effect” eventually terminates after a total time T , having reached a final amount of dissipative energy and having effected a total spatial extension S . The quantity S is called by BTW the “size” and the quantity T the “lifetime” of the avalanche—named by BTW a “cluster” (hereafter referred to as BTW cluster or BTW avalanche). As we shall see in more detail later on, it is of considerable importance to note that a BTW cluster *cannot*, and *should not*, be identified with a cluster in the usual sense. It is an avalanche, *not* a *static* object with a fixed structure which remains unchanged until it decays after a time interval (known as the lifetime in the usual sense).

In fact, it has been shown [13,14] that the distribution (D_S) of the “size” (which is a measure of the dissipative energy, S) and the distribution (D_T) of the lifetime (T) of BTW clusters in such open, dynamical, complex systems obey power laws:

$$D_S(S) \sim S^{-\mu}, \quad (1)$$

$$D_T(T) \sim T^{-\nu}, \quad (2)$$

where μ and ν are positive real constants. Such spatial and temporal power-law scaling behaviors can be, and have been, considered as the universal signals—the “fingerprints”—of the locally perturbed self-organized critical states in such systems. It is expected [13,14] that the general concept of self-organized criticality (SOC), which is complementary to chaos, may be *the* underlying concept for temporal and spatial scaling in a wide class of *open, nonequilibrium, complex systems*—although it is not yet known how the exponents of such power laws can be calculated analytically from fundamental theories such as that for gravitation or that for electromagnetism.

SOC has been *observed experimentally* in a large number of open, dynamical, complex systems in nonequilibrium [13–18], among which the following examples are of particular interest, because they illuminate several aspects of SOC which are relevant for the discussion in this paper.

First, the well-known Gutenberg-Richter law [19,15] for earthquakes as a special case of Eq. (1): In this case, earthquakes are BTW clusters due to SOC. Here, S stands for the released energy (the magnitude) of the observed earthquakes. $D_S(S)$ is the number of earthquakes at which an energy S is released. While details about the range of validity and the implications of this remarkable regularity can be found in Refs. [14,15,19], we wish to emphasize that the power-law behavior given by the Gutenberg-Richter law implies in particular the following. The question as to how large is a *typical* earthquake does not make sense.

Second, the sandpile (and ricepile) experiments [13,14] which show the simple regularities mentioned in Eqs. (1) and (2): In this example, we see how local perturbation can be caused by the addition of one grain of sand (note that we are dealing with an open system). Here, we can also see how the propagation of perturbation in form of the “domino effect” takes place, and develops into BTW clusters or avalanches of all possible sizes and durations. The size and duration distributions are given by Eqs. (1) and (2), respectively. This example is indeed a very attractive one, not only because such *experiments* can be, and have been, performed in *laboratories* [14], but also because they can be readily simulated on a PC [13,14].

Furthermore, it has been pointed out, and demonstrated by simple models [14,16–18], that the concept of SOC can also be applied to the biological sciences. It is amazing to see how phenomena as complicated as life and evolution can be simulated by simple models such as the “game of life” [16] and the “evolution model” [17,18].

Having seen that systems of interacting soft gluons are open, dynamical, complex systems, and that a wide class of open, dynamical, complex systems in the macroscopic world evolve into self-organized critical states which lead to fluctuations extending over all length and time scales, it seems natural to ask the following: Can such states and such fluctuations also exist in the microscopic world—on the level of quarks and gluons? In particular, can SOC be the dynamical origin of color-singlet gluon clusters which play the dominating role in inelastic diffractive scattering processes?

III. SOC IN INELASTIC DIFFRACTIVE SCATTERING PROCESSES?

One of the main goals of the present paper is to answer the questions mentioned at the end of the last section. We discuss, in this and in the following four sections, *how to look for* signals of SOC in systems of interacting gluons, and *what can we see* when we look for signals of SOC in scattering processes in which systems of interacting gluons play the dominating role.

Here, we explicitly see (a) the fundamental properties of the gluons, (b) the necessary conditions for the occurrence of SOC, and (c) the available technical possibilities strongly suggesting that the most favorable place to study the possible existence of such signals is (i) to look at the experimental results obtained in events associated with large rapidity gaps in deep-inelastic scattering, (ii) to look at the experimental results in inelastic diffractive hadron-hadron scattering, and (iii) to compare such observations with each other.

Having the special role played by “the colorless objects” in inelastic diffractive scattering in mind, let us begin our discussion with the following question: What *are* such “colorless objects”? Up to now, we do not know much about such objects. We know that they carry neither color nor any flavor quantum numbers. We know that they exist in high-energy reactions where soft gluons play the dominating role. We know that they can be probed in diffractive scattering processes, in the sense that they can interact with different beam particles. But there is a lot more which we do not know. For example, what is the mass of a typical “colorless object”? What is the lifetime of a typical “colorless object”? Do such objects have distinct electromagnetic structures? Are they hadron like? Before more and better empirical facts about such objects become available, guesses and/or speculations may be helpful, provided that they agree with the existing data and they are consistent with fundamental theoretical knowledge—in particular consistent with the basic properties of the gluons (the direct gluon-gluon coupling prescribed by the QCD Lagrangian, the confinement, and the nonconservation of gluon number, etc.). In this sense, we may wish to ask the following: Is it possible that the colorless objects are BTW clusters which exist due to SOC in systems of interacting soft gluons? We are aware of the fact that the existence of SOC cannot (at least cannot yet) be derived from a basic theory such as QCD [perhaps this can be and/or should be compared with the fact that the Gutenberg-Richter law for earthquakes cannot (at least cannot yet) be derived from gravitational theory]. But as in the case of earthquakes or any other open dynamical systems which leads to SOC, we can and we should ask the following: Can this be checked experimentally? Can this be done by looking for characteristic properties of SOC—in particular the SOC fingerprints mentioned in Eqs. (1) and (2) in the relevant experiments?

To answer these questions, it is useful to recall the following: Since the “colorless objects” are color singlets which can exist inside and/or outside the proton, the interactions between such color singlets as well as those between such objects and “the mother proton” should be of van der

Waals type. Hence, it is expected that such a colorless object can be readily separated as an entire object from the mother proton in scattering processes in which the momentum transfer is sufficient to overcome the binding energy due to the van der Waals type of interactions. This means that in inelastic diffractive scattering the beam particle (which is the virtual photon γ^* in DIS) should have a chance to encounter one of the color-singlet gluon clusters. For the reasons mentioned above, the struck colorless object can simply be “knocked out” and/or “carried away” by the beam particle in such a collision event. Hence, it seems that the question of whether “the colorless objects” are indeed BTW clusters is something that can be answered experimentally. In this connection we recall that, according to the general theory of SOC [13,14], the size of a BTW cluster is characterized by its dissipative energy, and in the case of systems of interacting soft gluons associated with the proton, the dissipative energy carried by the BTW cluster should be proportional to the energy fraction (x_P) carried by the colorless object. Hence, if the colorless object can indeed be considered as a BTW cluster due to SOC, we should be able to obtain information about the size distribution of such color-singlet gluon clusters by examining the x_P distributions of LRG events in the small- x_B region of DIS.

Having this in mind, we now take a closer look at the measured [3] “diffractive structure function” $F_2^{D(3)}(\beta, Q^2; x_P) \equiv \int dt F_2^{D(4)}(\beta, Q^2; x_P, t)$. Here, we note that $F_2^{D(4)}(\beta, Q^2; x_P, t)$ is related [3,4,8–10] to the differential cross section for large-rapidity-gap events,

$$\frac{d^4 \sigma^D}{d\beta dQ^2 dx_P dt} = \frac{4\pi\alpha^2}{\beta Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2^{D(4)}(\beta, Q^2; x_P, t), \quad (3)$$

in analogy to the relationship between the corresponding quantities [namely, $d^2\sigma/(dx_B dQ^2)$ and $F_2(x_B, Q^2)$] for normal deep-inelastic electron-proton scattering events:

$$\frac{d^2 \sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2}{x_B Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2(x_B, Q^2). \quad (4)$$

The kinematical variables, in particular β , Q^2 , x_P , and x_B (in both cases), are directly measurable quantities, the definitions of which are shown in Fig. 1 together with the corresponding diagrams of the scattering processes. We note that, although these variables are Lorentz invariants, it is sometimes convenient to interpret them in a “fast moving frame,” for example, the electron-proton center-of-mass frame where the proton’s three-momentum \vec{P} is large [i.e., its magnitude $|\vec{P}|$ and thus the energy $P^0 \equiv (|\vec{P}|^2 + M^2)^{1/2}$ are much larger than the proton mass M]. While Q^2 characterizes the virtuality of the spacelike photon γ^* , x_B can be interpreted, in such a “fast moving frame” (in the framework of the celebrated parton model), as the fraction of proton’s energy P^0 (or longitudinal momentum $|\vec{P}|$) carried by the struck charged constituent.

We recall that in the framework of the parton model, $F_2(x_B, Q^2)/x_B$ for “normal events” can be interpreted as the

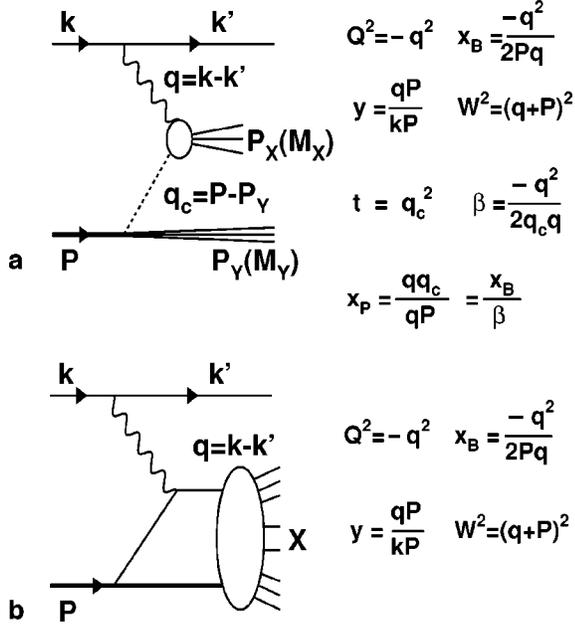


FIG. 1. The well-known Feynman diagrams (a) for diffractive and (b) for normal deep-inelastic electron-proton scattering are shown together with the relevant kinematical variables which describe such processes.

sum of the probability densities for the above-mentioned γ^* to interact with a charged constituent of the proton. In analogy to this, the quantity $F_2^{D(3)}(\beta, Q^2; x_p)/\beta$ for LRG events can be interpreted as the sum of the probability densities for γ^* to interact with a charged constituent which carries a fraction $\beta \equiv x_B/x_p$ of the energy (or longitudinal momentum) of the colorless object, under the condition that the colorless object (which we associate with a system of interacting soft gluons) carries a fraction x_p of proton's energy (or longitudinal momentum). We hereafter denote this charged-neutral and color-neutral gluon system by c_0^* (in Regge pole models [8] this object is known as the ‘‘Pomeron’’). Hence, by comparing Eq. (3) with Eq. (4) and by comparing the two diagrams shown in Fig. 1(a) and Fig. 1(b), it is tempting to draw the following conclusions.

The diffractive process is nothing else but a process in which the virtual photon γ^* encounters a c_0^* , and β is nothing else but the Bjorken variable with respect to c_0^* (this is why it is called x_{BC} in Ref. [11]). This means that a diffractive e^-p scattering event can be envisaged as an event in which the virtual photon γ^* collides with ‘‘a c_0^* target’’ instead of ‘‘the proton target.’’ Furthermore, since c_0^* is charge neutral, and a photon can only directly interact with an object which has electric charges and/or magnetic moments, it is tempting to assign c_0^* an electromagnetic structure function $F_2^c(\beta, Q^2)$, and study the interactions between the virtual photon and the quark(s) and antiquark(s) inside c_0^* . In such a picture (which should be formally the same as that of Regge pole models [8], if we would replace the c_0^* 's by ‘‘Pomerons’’) we are, however, confronted with the following two questions.

First, is it possible and meaningful to discuss the x_p dis-

tributions of the c_0^* 's without knowing the intrinsic properties, in particular the electromagnetic structures, of such objects?

Second, are the c_0^* 's hadron like, such that the electromagnetic structure of a (a typical, or an average) c_0^* can be studied in the same way as those for ordinary hadrons?

Since we wish to begin the quantitative discussion with something familiar to most of the readers in this community, and we wish to differentiate between the conventional approach and the SOC approach, we would like to discuss the second question here, and leave the first question to the next section. We recall that (see in particular the last two papers in Ref. [8]) in order to see whether the second question can be answered in the affirmative, we need to know whether $F_2^{D(3)}(\beta, Q^2; x_p)$ can be factorized in the form

$$F_2^{D(3)}(\beta, Q^2; x_p) = f_c(x_p) F_2^c(\beta, Q^2). \quad (5)$$

Here, $f_c(x_p)$ plays the role of a ‘‘kinematical factor’’ associated with the ‘‘target c_0^* ,’’ and x_p is the fraction of proton's energy (or longitudinal momentum) carried by c_0^* . [We could call $f_c(x_p)$ ‘‘the c_0^* flux,’’—in exactly the same manner as in Regge pole models [8], where it is called ‘‘the Pomeron flux.’’] $F_2^c(\beta, Q^2)$ is ‘‘the electromagnetic structure function of c_0^* ,’’ [the counterpart of $F_2(x_B, Q^2)$ of the proton] which—in analogy to the proton (or any other hadron)—can be expressed as

$$\frac{F_2^c(\beta, Q^2)}{\beta} = \sum_i e_i^2 [q_i^c(\beta, Q^2) + \bar{q}_i^c(\beta, Q^2)], \quad (6)$$

where q_i^c (\bar{q}_i^c) stands for the probability density for γ^* to interact with a quark (antiquark) of flavor i and electric charge e_i which carries a fraction β of the energy (or longitudinal momentum) of c_0^* . It is clear that Eq. (6) should be valid for all x_p values in this kinematical region; that is, both the right- and left-hand sides of Eq. (6) should be independent of the energy (momentum) carried by the ‘‘hadron’’ c_0^* .

Hence, to find out experimentally whether the second question can be answered in the affirmative, we only need to check whether the data are in agreement with the assumption that $F_2^c(\beta, Q^2)$ prescribed by Eqs. (5) and (6) exists. For such a test, we take the existing data [3] and plot $\log[F_2^{D(3)}(\beta, Q^2; x_p)/\beta]$ against $\log \beta$ for different x_p values. We note that under the assumption that the factorization shown in Eq. (5) is valid, the β dependence for a given Q^2 in such a plot should have exactly the same form as that in the corresponding $\log[F_2^c(\beta, Q^2)/\beta]$ vs $\log \beta$ plot, and that the latter is the analogue of the $\log[F_2(x_B, Q^2)/x_B]$ vs $\log x_B$ plot for normal events. In Fig. 2 we show the result of such plots for three fixed Q^2 values (3.5, 20, and 65 GeV², as representatives of three different ranges in Q^2). Our goal is to examine whether or how the β dependence of the function given in Eq. (6) changes with x_p . In principle, if there were enough data points, we should, and we could, do such a plot for the data sets associated with every x_p value. But unfortunately there are not so many data at present. What we can do, however, is to consider the β distributions in different x_p bins,

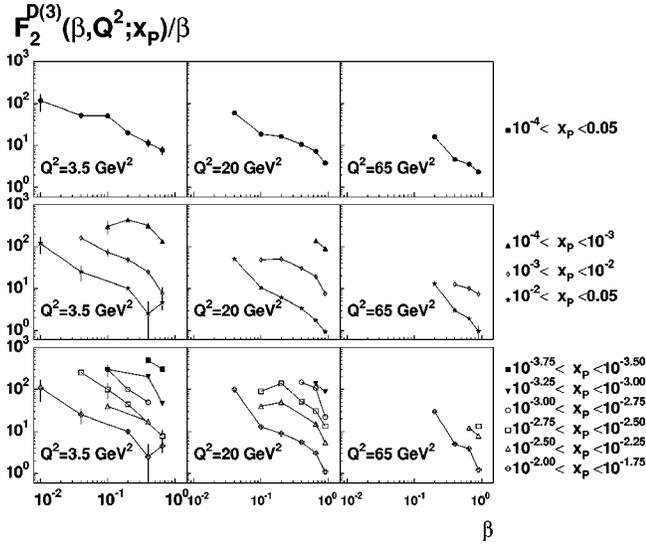


FIG. 2. $F_2^{D(3)}(\beta, Q^2; x_p)/\beta$ is plotted as a function of β for given x_p intervals and for fixed Q^2 values. The data are taken from Ref. [3]. The lines are only to guide the eye.

and to vary the bin size of x_p , so that we can explicitly see whether and how the shapes of the β distributions change. The results are shown in Fig. 2. The β distribution in the first row corresponds to the integrated value $\bar{F}_2^D(\beta, Q^2)$ shown in the literature [3,9]. Those in the second and third rows are obtained by considering different bins and/or by varying the sizes of the bins. By joining the points associated with a given x_p interval in a plot for a given Q^2 , we obtain the β distribution for a c_0^* carrying approximately the amount of energy $x_p P^0$, encountered by a photon of virtuality Q^2 . Taken together with Eq. (6) we can then extract the distributions $q_i^c(\beta, Q^2)$ and $\bar{q}_i^c(\beta, Q^2)$ for this Q^2 value, provided that $F_2^c(\beta, Q^2)/\beta$ is independent of x_p . But as we can see in Fig. 2, the existing data [3] show that the x_p dependence of this function is far from being negligible. Note in particular that according to Eq. (5), by choosing a suitable $f_p(x_p)$, we can shift the curves for different x_p values in the vertical direction (in this log-log plot), but *we can never change the shapes of the β distributions* which are different for different x_p values.

In order to see, and to realize, the meaning of the x_p dependence of the distributions of the charged constituents of c_0^* expressed in terms of $F_2^c(\beta, Q^2)/\beta$ in LRG events [see Eqs. (5) and (6)], let us, for a moment, consider normal deep-inelastic scattering events in the x_B region where quarks dominate ($x_B > 0.1$, say). Here we can plot the data for $\log[F_2(x_B, Q^2)/x_B]$ as a function of $\log x_B$ obtained at *different incident energies* (P^0 's) of the proton. *Suppose* we see that at a given Q^2 the data for x_B distributions taken at different values of P^0 are very much different. *Would* it still be possible to introduce $F_2(x_B, Q^2)$ as ‘‘the electromagnetic structure function’’ of the proton, from which we can extract the x_B distribution of the quarks $q_i(x_B, Q^2)$ at a given Q^2 ? The fact that it is not possible to assign an x_p -independent structure function $F_2^c(\beta, Q^2)/\beta$ to c_0^* which stands for the ‘‘Pomeron’’ and whose ‘‘flux’’ $f_c(x_p)$ is expected to be in-

dependent of β and Q^2 deserves to be taken seriously. It strongly suggests that the following picture *cannot* be true: ‘‘There exists a universal colorless object (call it Pomeron or c_0^* or something else) the exchange of which describes diffractive scattering in general and DIS off protons in particular. This object is hadron like in the sense that it has not only a typical size and a typical lifetime, but also a typical electromagnetic structure which can, e.g., be measured and described by an electromagnetic structure function.’’

In summary of this section, we note that the empirical facts mentioned above show that *no* energy-independent electromagnetic structure function can be assigned to the expected universal colorless object c_0^* . This experimental fact is of considerable importance, because it is the first indication that, if there is a *universal* ‘‘colorless object,’’ this object *cannot* be considered as an ordinary hadron. It has to be something else. In fact, as we shall see below, this property is closely related to the observation that such an object *cannot* have a typical size or a typical lifetime. To be more precise, the fact that the data [3] *cannot* accommodate the simple factorization assumption shown in Eq. (5), in which a universal Pomeron flux with a unique hadronlike Pomeron structure function exists, can be considered as an important support for the proposed SOC picture because a BTW cluster, which has neither a typical size nor a typical lifetime, *cannot* have a universal static structure. With these characteristic properties of the colorless objects in mind, one may view this as [20] an overlap between the SOC picture and the partonic picture for Pomeron and/or Pomeron and Reggeon [8,20] in which, beside the Pomeron, exchange of (in general an infinite number of) subleading trajectories is possible. In fact, it has been reported [3,4] that very good agreement can be achieved between the data [3,4] and these types of models. Hence, in order to differentiate between the two approaches, it is not only useful but also necessary to examine the corresponding predictions for the dependence on the invariant momentum transfer t . This will be discussed in detail in Secs. VIII–XI.

IV. DISTRIBUTIONS OF THE GLUON CLUSTERS

After having seen that the existing data do not allow us to assign an energy-independent electromagnetic structure function to ‘‘the colorless object’’ such that the universal colorless object (c_0^*) can be treated as an ordinary hadron, let us now come back to the first question in Sec. III, and try to find out whether it is never the less possible, and meaningful, to talk about the x_p distribution of c_0^* . As we shall see in this section, the answer to this question is yes. Furthermore, we shall also see that in order to answer this question in the affirmative, we *do not* need the factorization mentioned in Eq. (5), and we *do not* need to know whether the c_0^* 's are hadron like. But as we have already mentioned above, it is of considerable importance to discuss the second question so that we can understand the origin and the nature of the c_0^* 's.

In view of the fact that we do use the concept ‘‘distributions of gluons’’ in deep-inelastic lepton-hadron scattering, although the gluons do not directly interact with the virtual photons, we shall introduce the notion ‘‘distribution of c_0^* ’’

in a similar manner. In order to see what we should do for the introduction of such distributions, let us recall the following.

For normal deep-inelastic e^-p collision events, the structure function $F_2(x_B, Q^2)$ can be expressed in terms of the distributions of partons, where the partons are not only quarks and antiquarks, but also gluons which can contribute to the structure function by quark-antiquark pair creation and annihilation. In fact, in order to satisfy energy-momentum conservation (in the electron-proton system), the contribution of the gluons $x_g g(x_g, Q^2)$ has to be taken into account in the energy-momentum sum rule for all measured Q^2 values. Here, we denote by $g(x_g, Q^2)$ the probability density for the virtual photon γ^* (with virtuality Q^2) to meet a gluon which carries the energy [momentum] fraction x_g of the proton, analogous to $q_i(x_B, Q^2)$ [or $\bar{q}_i(x_B, Q^2)$] which stands for the probability density for this γ^* to interact with a quark [or an antiquark] of flavor i and electric charge e_i which carries the energy [momentum] fraction x_B of the proton. We note that while both x_B and x_g stand for energy [or longitudinal momentum] fractions carried by partons, the former can be, but the latter *cannot* be, directly measured.

Having these, in particular the energy-momentum sum rule, in mind, we immediately see the following: In a given kinematical region in which the contributions of only one category of partons (for example, quarks for $x_B > 0.1$ or gluons for $x_B < 10^{-2}$) dominate, the structure function $F_2(x_B, Q^2)$ can approximately be related to the distributions of that particular kind of partons in a very simply manner. In fact, the expressions below can be, and have been, interpreted as the probability densities for the virtual photon γ^* (with virtuality Q^2) to meet a quark or a gluon which carries the energy (momentum) fraction x_B or x_g , respectively;

$$\frac{F_2(x_B, Q^2)}{x_B} \approx \sum_i e_i^2 q_i(x_B, Q^2) \quad (7a)$$

or

$$\frac{F_2(x_B, Q^2)}{x_g} \approx g(x_g, Q^2). \quad (7b)$$

The relationship between $q_i(x_B, Q^2)$, $g(x_g, Q^2)$, and $F_2(x_B, Q^2)$ as they stand in Eqs. (7a) and (7b) are general and formal (this is the case especially for that between g and F_2) in the following sense: Both $q_i(x_B, Q^2)$ and $g(x_g, Q^2)$ contribute to the energy-momentum sum rule and both of them are in accordance with the assumption that partons of a given category (quarks or gluons) dominate a given kinematical region (here $x_B > 0.1$ and $x_B < 10^{-2}$, respectively). But neither the dynamics which leads to the observed Q^2 dependence nor the relationship between x_g and x_B is given. This means that *without further theoretical inputs*, the simple expression for $g(x_g, Q^2)$ as given by Eq. (7b) is *practically useless*.

Having learned this, we now discuss what happens if we assume, in diffractive lepton-nucleon scattering, that the color-singlet gluon clusters (c_0^* 's) dominate the small- x_B re-

gion ($x_B < 10^{-2}$, say). In this simple picture, we are assuming that the following is approximately true: The gluons in this region appear predominantly in the form of c_0^* 's. The interaction between the struck c_0^* and the rest of the proton can be neglected during the γ - c_0^* collision such that we can apply an impulse approximation to the c_0^* 's in this kinematical region. That is, here we can introduce—in the same manner as we do for other partons [see Eqs. (7)], a probability density $D_S(x_P|\beta, Q^2)$ for γ^* in the diffractive scattering process to “meet” a c_0^* which carries the fraction x_P of the proton's energy $P^0 = (|\vec{P}|^2 + M^2)^{1/2} \approx |\vec{P}|$ (where \vec{P} is the momentum and M is the mass of the proton). In other words, in diffractive scattering events for processes in the kinematical region $x_B < 10^{-2}$, we should have, instead of $g(x_g, Q^2)$, the following:

$$\frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P} \approx D_S(x_P|\beta, Q^2). \quad (8)$$

Here, $x_P P^0$ is the energy carried by c_0^* , and β indicates the corresponding fraction carried by the struck charged constituent in c_0^* . In connection with the similarities and the differences between $q_i(x_B, Q^2)$, $g(x_B, Q^2)$ in Eqs. (7) and $D_S(x_P|\beta, Q^2)$ in Eq. (8), it is useful to note in particular the significant difference between x_g and x_P , and thus that between the x_g distribution $g(x_g, Q^2)$ of the gluons and the x_P distribution $D_S(x_P|\beta, Q^2)$ of the c_0^* 's: Both x_g and x_P are energy (longitudinal momentum) fractions of charge-neutral objects, with which γ^* *cannot* directly interact. But in contrast to x_g , x_P *can be directly measured in experiments*, namely, by making use of the kinematical relation

$$x_P \approx \frac{Q^2 + M_x^2}{Q^2 + W^2}, \quad (9)$$

and by measuring the quantities Q^2 , M_x^2 , and W^2 in every collision event. Here, Q , M_x , and W stand, respectively, for the invariant momentum transfer from the incident electron, the invariant mass of the final hadronic state after the γ^* - c_0^* collision, and the invariant mass of the entire hadronic system in the collision between γ^* and the proton. Note that $x_B \equiv \beta x_P$; hence β is also measurable. This means that, in sharp contrast to $g(x_g, Q^2)$, *experimental information* on $D_S(x_P|\beta, Q^2)$ in particular its x_P dependence can be obtained—*without further theoretical inputs*.

V. FIRST SOC FINGERPRINT: SPATIAL SCALING

We mentioned at the beginning of Sec. III that in order to find out whether the concept of SOC indeed plays a role in diffractive DIS we need to check the fingerprints of SOC shown in Sec. II, and that such tests can be made by examining the corresponding cluster distributions obtained from experimental data. We are now ready to do this, because we have learned in Secs. III and IV that it is not only meaningful but also possible to extract x_P distributions from the measured diffractive structure functions, although the c_0^* 's *can-*

not be treated as hadrons. In fact, as we can explicitly see in Eqs. (8) and (9), in order to extract the x_P dependence of the c_0^* 's from the data, detailed knowledge about the intrinsic structure of the c_0^* 's is not necessary.

Having these in mind, we now consider D_S as a function of x_P for given values of β and Q^2 , and plot $F_2^{D(3)}(\beta, Q^2; x_P)/x_P$ against x_P for different sets of β and Q^2 . The results of such log-log plots are shown in Fig. 3. As we can see, the data [3,4] suggest that the probability density for the virtual photon γ^* to meet a color-neutral and charged-neutral object c_0^* with energy (longitudinal momentum) fraction x_P has a power law behavior in x_P , and the exponent of this power law depends very little on Q^2 and β . This is to be compared with $D_S(S)$ in Eq. (1), where S , the dissipative energy (the size of the BTW cluster), corresponds to the energy of the system c_0^* . The latter is $x_P P^0$, where P^0 is the total energy of the proton.

It means that the existing data [3,4] show that $D_S(x_P|\beta, Q^2)$ exhibits the same kind of power-law behavior as the size distribution of BTW clusters. This result is in accordance with the expectation that self-organized critical phenomena may exist in systems of interacting soft gluons in diffractive deep-inelastic electron-proton scattering processes.

We note that, up to now, we have only argued (in Sec. I) that such gluon systems are open, dynamical, complex systems in which SOC may occur, and we have mentioned (in Sec. II) the ubiquitousness of SOC in nature. Having seen the experimental evidence that one of the ‘‘SOC fingerprints’’ (which are necessary conditions for the existence of SOC) indeed exists, let us now take a second look at such gluon systems from a theoretical standpoint. Viewed from a ‘‘fast moving frame’’ which can, for example, be the electron-proton c.m. system (c.m.s.) frame, such systems of interacting soft gluons are part of the proton (although color singlets can also be outside the confinement region). Soft gluons can be intermittently emitted or absorbed by gluons in such a system, as well as by gluons, quarks, and antiquarks outside the system. The emission and absorption processes are due to local interactions prescribed by the well-known QCD Lagrangian (here ‘‘the running coupling constants’’ are in general large, because the distances between the interacting colored objects cannot be considered as ‘‘short’’; remember that the spatial dimension of a c_0^* can be much larger than that of a hadron). In this connection, it is useful to keep the following in mind: As a result of the complexity of the system, details about the local interactions may be relatively unimportant, while general and/or global features—for example, energy flow between different parts (neighbors and neighbor’s neighbors, . . .) of the system—may play an important role.

How far can one go in neglecting dynamical details when one deals with such open complex systems? In order to see this, let us recall how Bak and Sneppen [17] succeeded in modeling some of the essential aspects of the evolution in nature. They consider the ‘‘fitness’’ of different ‘‘species,’’ related to one another through a ‘‘food chain,’’ and assumed that the species with the lowest fitness is most likely to dis-

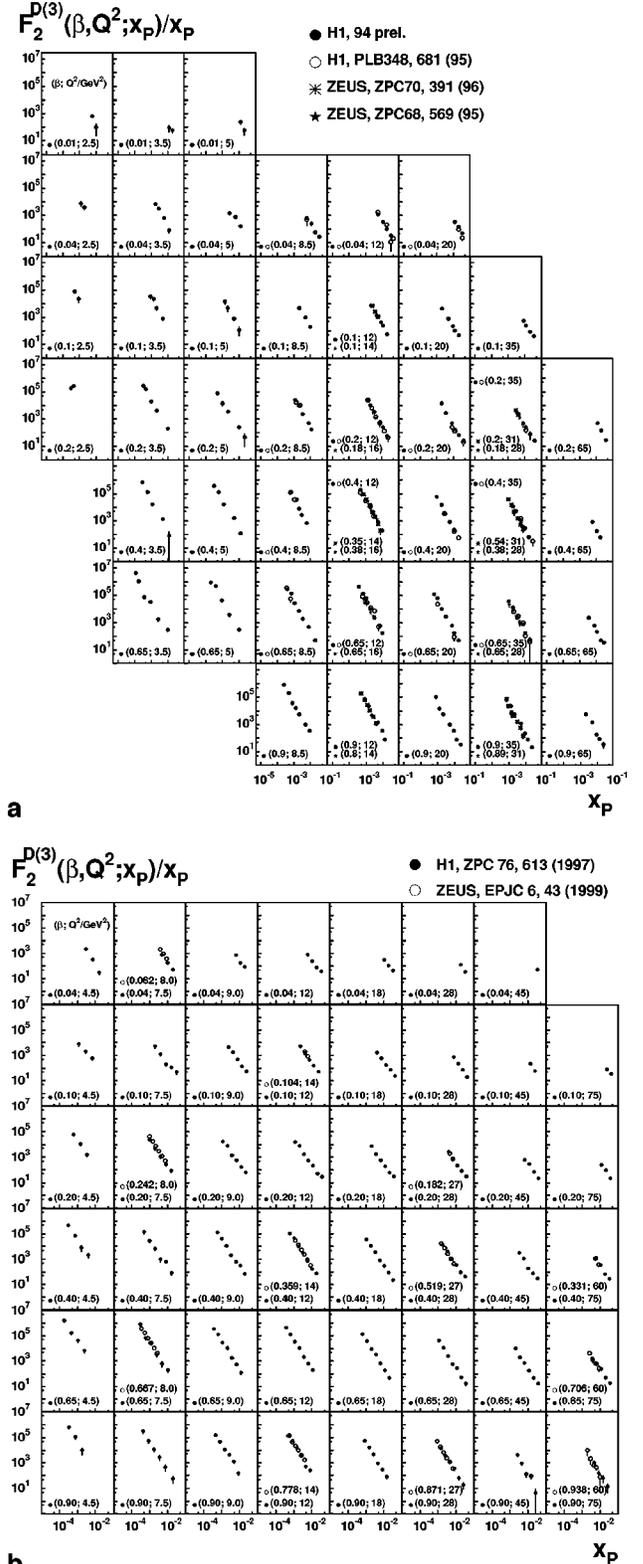


FIG. 3. (a) $F_2^{D(3)}(\beta, Q^2; x_P)/x_P$ is plotted as a function of x_P for different values of β and Q^2 . The data are taken from Ref. [3]. (b) Same as (a) but with data taken from Ref. [4]. Note that in these log-log plots, almost all existing data points lie on straight lines with approximately the *same* slope irrespective of the values of Q^2 and/or β .

appear or mutate at the next time step in their computer simulations. The crucial step in their simulations that *drives* evolution is the adaption of the individual species to its present *environment* (neighborhood) through mutation and selection of a fitter variant. Other interacting species form part of the *environment*. This means that the neighbors will be influenced by every time step. The result these authors obtained strongly suggests that the process of evolution is a self-organized critical phenomenon. One of the essential simplifications they made in their evolution models [17,18] is the following: Instead of the explicit connection between the fitness and the configuration of the genetic codes, they use random numbers for the fitness of the species. Furthermore, as they have pointed out in their papers, they could in principle have chosen to model evolution on a less coarse-grained scale by considering mutations at the individual level rather than on the level of species, but that would make the computation prohibitively difficult.

Having these in mind, we are naturally led to the following questions: Can we consider the creation and annihilation processes of colorless systems of interacting soft gluons associated with a proton as “evolution” in a microscopic world? Before we try to build models for a quantitative description of the data, can we simply apply the existing evolution models [17,18] to such open, dynamical, complex systems of interacting soft gluons and check whether some of the essential features of such systems can be reproduced?

To answer these questions, we now report on the result of our first trial in this direction: Based on the fact that we know *very little* about the detailed reaction mechanisms in such gluon systems and *practically nothing* about their structures, we simply *ignore* them, and assume that they are self-similar in space [this means that color-singlet gluon clusters (c_0^*) can be considered as clusters of c_0^* 's and so on]. Next, we divide them into an arbitrarily given number of subsystems s_i (which may or may not have the same size). Such a system is open, in the sense that neither its energy ε_i nor its gluon number n_i has a fixed value. Since we do not know, in particular, how large the ε_i 's are, we use random numbers. As far as the n_i 's are concerned, since we do not know how these numbers are associated with the energies in the subsystems s_i , except that they are not conserved quantities, we just ignore them, and consider only the ε_i 's. As in Ref. [17] or in Ref. [18], the random number of this subsystem as well as those of the fixed [17] or random (see the first paper of Ref. [18]) neighbors will be changed at every time step. Note that this is how we simulate the processes of energy flow due to the exchange of gluons between subsystems, as well as those with gluons, quarks, and antiquarks outside the system. In other words, in the spirit of Bak and Sneppen [17] we are neglecting the dynamical details *totally*. Having in mind that, in such systems, the gluons as well as the subsystems (s_i 's) of gluons are *virtual* (space like), we can ask the following: How long can such a colorless subsystem s_i of interacting soft gluons exist, which carries energy ε_i ? According to the uncertainty principle, the answer should be the following: The time interval in which the subsystem s_i can exist is proportional to $1/\varepsilon_i$, and this quantity can be considered as the lifetime τ_i of s_i . In this sense, those colorless subsystems

of gluons are expected to have larger probabilities to mutate when they are associated with higher energies and thus shorter “lifetimes.” Note that the basic local interaction in this self-organized evolution process is the emission (or absorption) of gluons by gluons prescribed by the QCD Lagrangian—although the detailed mechanisms (which can in principle be explicitly written down by using the QCD Lagrangian) do not play a significant role.

In terms of the evolution model [17,18] we now call s_i the “species” and identify the corresponding lifetime τ_i as the “fitness of s_i .” Because of the one-to-one correspondence between τ_i and ε_i , where the latter is a random number, we can also directly assign random numbers to the τ_i 's instead. From now we can adopt the evolution model [17,18] and note that, at the start of such a process (a simulation), the fitness on average grows, because the least fit are always eliminated. Eventually the fitness does not grow any further on average. All gluons have a fitness above some threshold. At the next step, the least fit species (i.e., the most energetic subsystem s_i of interacting soft gluons), which would be right at the threshold, will be “replaced” and starts an avalanche (or punctuation of mutation events), which is causally connected with this triggering “replacement.” After a while, the avalanche will stop, when all the fitnesses again will be over that threshold. In this sense, the evolution goes on, and on, and on. As in Refs. [17] and [18], we can monitor the duration of every avalanche, that is, the total number of mutation events in every one of them, and count how many avalanches of each size are observed. The avalanches mentioned here are special cases of those discussed in Sec. II. Their size and lifetime distributions are given by Eq. (1) and Eq. (2), respectively. Note in particular that the avalanches in the Bak-Sneppen model correspond to sets of subsystems s_i , the energies (ε_i) of which are too high “to be fit for the colorless systems of low-energy gluons.” It means that in the proposed picture, what the virtual photon in deep-inelastic electron-proton scattering “meet” are those a “less fit” one—those who carry “too much” energy. In a geometrical picture this means that it is more probable for such “relatively energetic” color-singlet gluon clusters (c_0^*) to be spatially further away from the (confinement region of) the proton.

There exists, in the mean time, already several versions of evolution models [14,18] based on the original idea of Bak and Sneppen [17]. Although SOC phenomena have been observed in all these cases [14,17,18], the slopes of the power-law distributions for the avalanches are different in different models—depending on the rules applied to the mutations. The values range from approximately -1 to approximately -2 . Furthermore, these models [14,17,18] seem to show that neither the size nor the dimension of the system used for the computer simulation plays a significant role.

Hence, if we identify the colorless charge-neutral object c_0^* encountered by the virtual photon γ^* with such an avalanche, we are identifying the lifetime of c_0^* with T and the “size” (that is, the total amount of dissipative energy in this “avalanche”) with the total amount of energy of c_0^* . Note that the latter is nothing else but $x_P P^0$, where P^0 is the total

energy of the proton. This is how and why the S distribution in Eq. (1) and the x_P distribution of $D_S(x_P|\beta, Q^2)$ in Eq. (8) are related to each other.

VI. SECOND FINGERPRINT: TEMPORAL SCALING

In this section we discuss in more detail the effects associated with the time degree of freedom. In this connection, some of the concepts and methods related to the two questions raised in Sec. III are of great interest. In particular, one may wish to know *why* the parton picture does not work equally well for hadrons and for c_0^* 's. The answer is very simple: The time degree of freedom cannot be ignored when we apply the impulse approximation, and the applicability of the latter is the basis of the parton model. We recall that, when we apply the parton model to stable hadrons, the quarks, antiquarks, and gluons are considered as free and stable objects, while the virtual photon γ^* is associated with a given interaction time $\tau_{\text{int}}(Q^2, x_B)$ characterized by the values Q^2 and x_B of such scattering processes. We note, however, that this is possible only when the interaction time τ_{int} is much shorter than the corresponding time scales related to hadron structure (in particular, the average propagation time of color interactions in hadrons). Having these in mind, we see that we are confronted with the following questions when we deal with c_0^* 's which have finite lifetimes: Can we consider the c_0^* 's as ‘‘free’’ and ‘‘stable’’ particles when their lifetimes are *shorter* than the interaction time $\tau_{\text{int}}(Q^2, x_B)$? Can we say that a $\gamma^*-c_0^*$ collision process takes place, in which the incident γ^* is absorbed by one or a system of the charged constituents of c_0^* , when the lifetime T of c_0^* is *shorter* than $\tau_{\text{int}}(Q^2, x_B)$?

Since the notion of ‘‘stable objects’’ or ‘‘unstable objects’’ depends on the scale which is used in the measurement, the question as to whether a c_0^* can be considered as a parton (in the sense that it can be considered as a free ‘‘stable object’’ during the $\gamma^*-c_0^*$ interaction) depends very much on the interaction time $\tau_{\text{int}}(Q^2, x_B)$. Here, for given values of Q^2 , x_B , and thus $\tau_{\text{int}}(Q^2, x_B)$, only those c_0^* 's whose lifetimes (T 's) are greater than $\tau_{\text{int}}(Q^2, x_B)$ can absorb the corresponding γ^* . That is to say, when we consider diffractive electron-proton scattering in kinematical regions in which c_0^* 's dominate, we must keep in mind that the measured cross sections (and thus the diffractive structure function $F_2^{D(3)}$) only include contributions from collision events in which the condition $T > \tau_{\text{int}}(Q^2, x_B)$ is satisfied.

We note that τ_{int} can be estimated by making use of the uncertainty principle. In fact, by calculating $1/q^0$ in the above-mentioned reference frame, we obtain

$$\tau_{\text{int}} = \frac{4|\vec{P}|}{Q^2} \frac{x_B}{1-x_B}, \quad (10)$$

which implies that, for given $|\vec{P}|$ and Q^2 values,

$$\tau_{\text{int}} \propto x_B, \quad \text{for } x_B \ll 1. \quad (11)$$

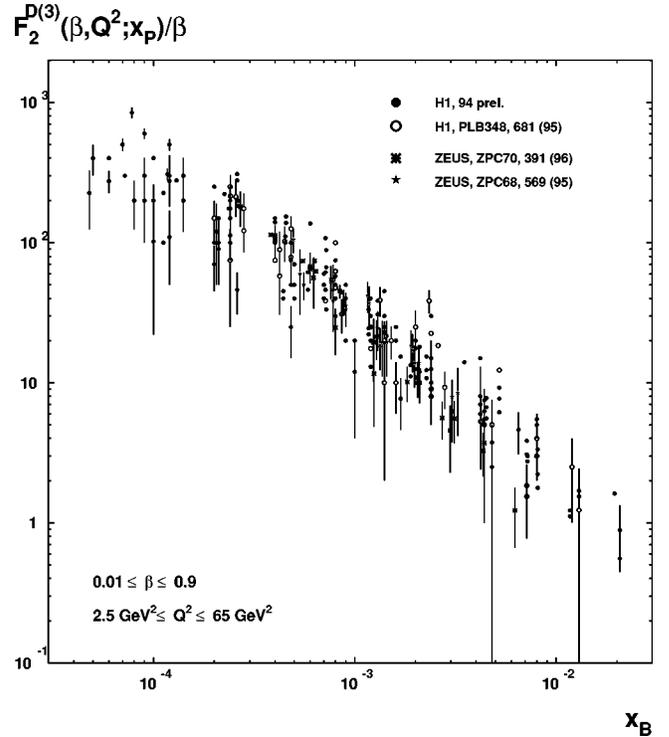


FIG. 4. $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ is plotted as a function of x_B in the indicated β and Q^2 ranges. The data are taken from Ref. [3].

This means that for diffractive e^-p scattering events in the small- x_B region at given $|\vec{P}|$ and Q^2 values, x_B is directly proportional to the interaction time τ_{int} . Taken together with the relationship between τ_{int} and the minimum lifetime $T(\text{min})$ of the c_0^* 's mentioned above, we reach the following conclusion: The distribution of this minimum value $T(\text{min})$ of the c_0^* 's which dominate the small- x_B ($x_B < 10^{-2}$, say) region can be obtained by examining the x_B dependence of $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ discussed in Eqs. (5), (6) and in Fig. 2. This is because, as a result of the fact that this function is proportional to the quark (antiquark) distributions $q_i^c(\bar{q}_i^c)$ which can be directly probed by the incident virtual photon γ^* , by measuring $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ as a function of $x_B \equiv \beta x_P$, we are in fact asking the following question: Do the distributions of the charged constituents of c_0^* depend on the interaction time τ_{int} and thus on the minimum lifetime $T(\text{min})$ of the to-be-detected c_0^* ? We use the identity $x_B \equiv \beta x_P$ and plot the quantity $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ against the variable x_B for fixed values of β and Q^2 . The result of such a log-log plot is given in Fig. 4. It shows not only how the dependence on the time degree of freedom can be extracted from the existing data [3], but also that, for all the measured values of β and Q^2 , the quantity

$$p(x_B|\beta, Q^2) \equiv \frac{F_2^{D(3)}(\beta, Q^2; x_B/\beta)}{\beta} \quad (12)$$

is approximately independent of β and independent on Q^2 . This strongly suggests that the quantity given in Eq. (12) is associated with some *global* features of c_0^* —consistent with

the observation made in Sec. III which shows that it *cannot* be used to describe the *structure* of c_0^* . This piece of empirical fact can be expressed by setting $p(x_B|\beta, Q^2) \approx p(x_B)$. By taking a closer look at this log-log plot, as well as the corresponding plots for different sets of fixed β and Q^2 values (such plots are not shown here; they are similar to those in Fig. 3), we see that they are straight lines, indicating that $p(x_B)$ obeys a power law. What does this piece of experimental fact tell us? What can we learn from the distribution of the lower limit of the lifetimes (of the gluon systems c_0^* 's)?

In order to answer these questions, let us, for a moment, assume that we know the lifetime distribution $D_T(T)$ of the c_0^* 's. In such a case, we can readily evaluate the integral

$$I[\tau_{\text{int}}(x_B)] \equiv \int_{\tau_{\text{int}}(x_B)}^{\infty} D_T(T) dT, \quad (13)$$

and thus obtain the number density of all those clusters which live longer than the interaction time $\tau_{\text{int}}(x_B)$. Hence, under the statistical assumption that the chance for a γ^* to be absorbed by one of those c_0^* 's of lifetime T is proportional to $D_T(T)$ [provided that $\tau_{\text{int}}(Q^2, x_B) \leq T$; otherwise this chance is zero], we can then interpret the integral in Eq. (13) as follows: $I[\tau_{\text{int}}(Q^2, x_B)] \propto p(x_B)$ is the probability density for γ^* [associated with the interaction time $\tau_{\text{int}}(x_B)$] to be absorbed by c_0^* 's. Hence,

$$D_T(x_B) \propto \frac{d}{dx_B} p(x_B). \quad (14)$$

This means in particular that the fact that $p(x_B)$ [introduced in Eq. (12)] obeys a power law in x_B implies that $D_T(T)$ obeys a power law in T . Such a *behavior is similar* to that shown in Eq. (2). In order to see the *quality* of this power-law behavior of D_T and the *quality* of its independence of Q^2 and β , we compare the above-mentioned behavior with the existing data [3,4]. In Fig. 5, we show the log-log plots of $d/dx_B[p(x_B)]$ against x_B . In doing this plot, we keep the definition of $p(x_B|\beta, Q^2)$ given in Eq. (12) and its weak β and Q^2 dependences in mind, and we note that $d/dx_B[p(x_B)]$ is approximately $F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B)$, provided that $p(x_B|\beta, Q^2)$ shows a power-law behavior in x_B . Here, we not only see that the quality of the power-law behavior of D_T in T is intimately related to the quality of the power-law behavior of $F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B)$ in x_B , but also how weak the β and Q^2 dependences are.

VII. SOC FINGERPRINTS IN INELASTIC DIFFRACTIVE γ^*p , γp , pp , AND $\bar{p}p$ SCATTERING PROCESSES

We have seen, in Secs. V and VI, that in diffractive deep-inelastic electron-proton scattering, the size and the lifetime distributions of the color-singlet gluon clusters (c_0^*) obey power laws, and that the exponents depend very little on the variables β and Q^2 . We interpreted the power-law behaviors as the fingerprints of SOC which are expected to manifest themselves in systems of interacting soft gluons (which play

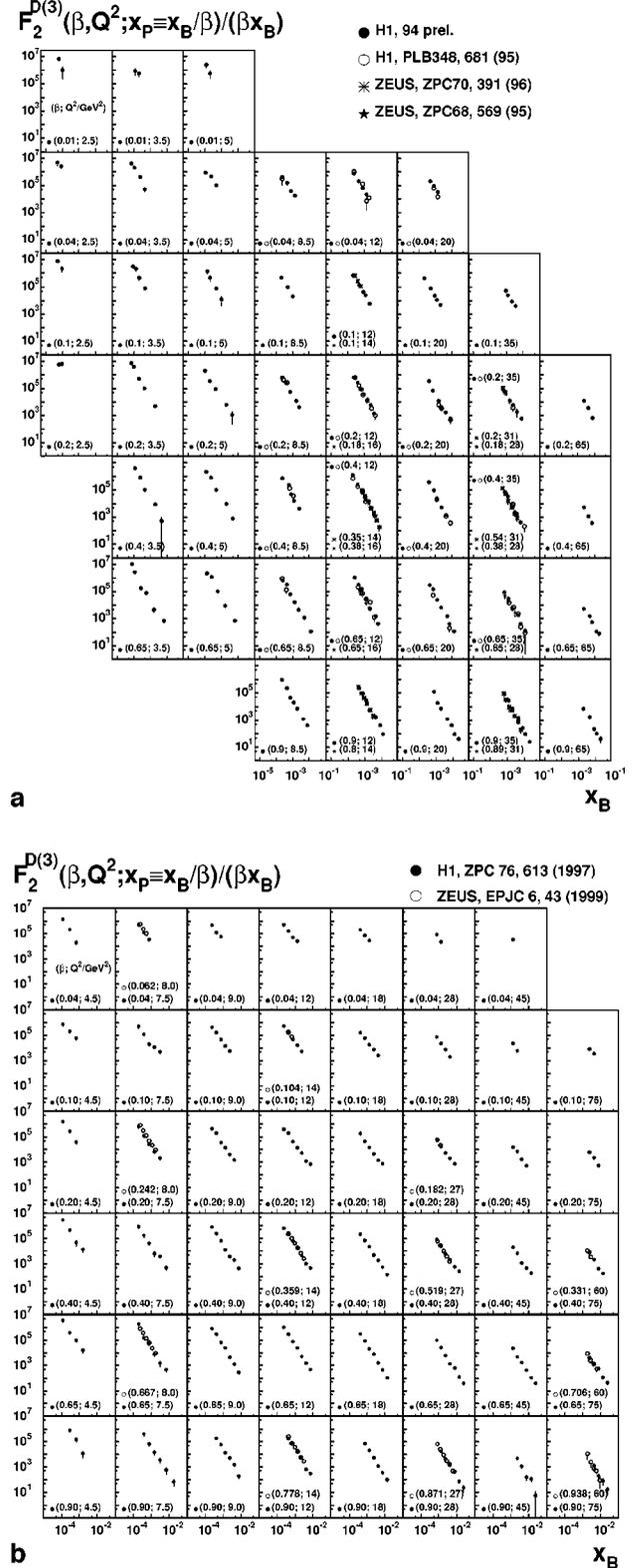


FIG. 5. (a) $F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B)$ is plotted as a function of x_B for fixed β and Q^2 values. The data are taken from Ref. [3]. (b) Same as (a) but with data taken from Ref. [4]. Note that in these log-log plots, almost all existing data points lie on straight lines with approximately the *same* slope irrespective of the values of Q^2 and/or β .

the dominating role in diffractive DIS). This expectation is based on the fact that the fundamental properties of gluons (in particular the direct gluon-gluon coupling, the confinement, and the nonconservation of gluon numbers) show that the necessary conditions for the existence of SOC in systems of interacting gluons are satisfied and the fact that (as we can see in various open dynamical complex systems) power-law behaviors in size and lifetime distributions are indeed reliable indicators for the existence of SOC. In this sense, the existence of such power-law behavior can be understood in terms of the QCD-based SOC picture, although it is not (at least not yet) possible to derive the power-law behavior of the size and lifetime distributions, and to calculate the exponents by using nonperturbative QCD. But can the observed approximate independence (or weak dependence) of the exponents on Q^2 and β also be understood in terms of the QCD-based SOC picture mentioned above? In particular, what do we expect to see in photoproduction processes where the associated value for Q^2 is approximately zero? We note that the possible relationship between the $Q^2 >$ a few GeV^2 case and the $Q^2 \approx 0$ case in diffractive scattering is of considerable interest for many reasons. One of them is the fact that, by comparing these two cases, we can see the fundamental difference between the conventional (pQCD-corrected parton model plus Regge phenomenology) picture and the proposed QCD-based SOC picture for inelastic diffractive scattering. In the conventional picture, the $Q^2 >$ a few GeV^2 case is “hard” and thus should be described by concepts and methods of parton model and pQCD, while the $Q^2 \approx 0$ case is “soft” and thus should be understood in terms of Regge poles. What are the predictions of the proposed SOC picture? What do the experimental data tell us in this connection? Would a systematic comparison of the existing data at different Q^2 values—including those near $Q^2 = 0$ —be useful in understanding the underlying reaction mechanism(s) of diffractive scattering in general and differentiate between the conventional and the proposed picture in particular?

In order to answer these questions, let us recall the space-time aspects of the collision processes which are closely related to the above-mentioned power-law behaviors. Viewed in a fast moving frame (e.g., the c.m.s. of the colliding electron and proton), the states of the interacting soft gluons originating from the proton are self-organized. The c_0^* caused by local perturbations and developed through “domino effects” are BTW avalanches (see Secs. I and V), the size distribution of which [see Eqs. (8) and (1)] is given by Fig. 3. This explicitly shows that there are c_0^* s of all sizes, because a power-law size distribution implies that there is no scale in size. Recall that, since such c_0^* s are color singlets, their spatial extensions can be much larger than that of the proton, and thus they can be “seen” also *outside* the proton by a virtual photon originating from the electron. In other words, what the virtual photon encounters is a cloud of c_0^* s, every one of which is in general partly inside and partly outside the proton.

The virtual photon, when it encounters a c_0^* , will be absorbed by the charged constituents (quarks and antiquarks

due to fluctuations of the gluons) of the gluon system. Here it is useful to recall that in such a space-time picture, Q^2 is inversely proportional to the transverse size, and x_B is a measure of the interaction time [see Eqs. (10) and (11) in Sec. VI] of the virtual photon. It is conceivable that the values for the cross sections for virtual photons (associated with a given Q^2 and a given x_B) to collide with c_0^* s (of a given size and a given lifetime) may depend on these variables. But since the processes of self-organization (which produce such c_0^* s) take place independent of the virtual photon (which originates from the incident electron and enters the “cloud” to look for suitable partners), the power-law behaviors of the size and lifetime distributions of the c_0^* s are expected to be independent of the properties associated with the virtual photon. This means that by using γ^* s associated with different values of Q^2 to detect c_0^* s of various sizes, we are moving up or down on the straight lines in the log-log plots for the size and lifetime distributions, the slopes of which do not change. In other words, the observed approximate Q^2 independence of the slope in the above-mentioned log-log plots of the data can be considered as a natural consequence of the QCD-based SOC picture.

As far as the β dependence is concerned, we recall the results obtained in Secs. III and IV, which explicitly show the following: The c_0^* s *cannot* be considered as *hadrons*. In particular, it is neither possible nor meaningful to talk about “the electromagnetic structure of the (or a typical, or an average) c_0^* .” This is not only because the power-law behavior of the size and the lifetime distributions of such c_0^* s implies that such objects—although they are color singlets—can have neither a typical (an average) size nor a typical (an average) lifetime, but also because of the following fact: When $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ which is usually known [9] as “the electromagnetic structure function of the colorless object exchanged in diffractive scattering processes” is plotted as functions of β (cf. Fig. 2), we see a rather significant x_P dependence. This *does not mean*, however, that the measured β dependence of $F_2^{D(3)}$ *cannot* provide us with *any* further information on the electromagnetic properties of the color-singlet gluon clusters (c_0^*). This is because the c_0^* s which play the dominating role in diffractive scattering are color singlets; hence, even when such clusters are BTW avalanches which have neither a typical size nor a typical lifetime, a set of such clusters with given size and lifetime distributions can nevertheless be considered as *a specific set of color-singlet gluon clusters with distinct properties*. Hence, when the β dependence of $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$ is examined in inelastic diffractive scattering processes, the electromagnetic properties of such a set of color-singlet gluon clusters are probed by the incident (virtual or real, depending on the Q^2 value of the event) photons. In this connection, it is perhaps useful to consider the β distribution integrated over x_P .

For the purpose of comparing SOC fingerprints obtained at different Q^2 values, we are interested much more in measurable quantities in which the integrations over β have been carried out. A suitable candidate for this purpose is the differential cross section

$$\begin{aligned} \frac{1}{x_P} \frac{d^2 \sigma^D}{dQ^2 dx_P} &= \int d\beta \frac{4\pi\alpha^2}{\beta Q^4} \left(1 - y + \frac{y^2}{2}\right) \frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P} \\ &\approx \int d\beta \frac{4\pi\alpha^2}{\beta Q^4} \left(1 - y + \frac{y^2}{2}\right) D_S(x_P | \beta, Q^2). \end{aligned} \quad (15)$$

Together with Eqs. (3) and (8), we see that this cross section is nothing else but the effective β -weighted x_P distribution $D_S(x_P | Q^2, \beta)$ of the c_0^* 's. Note that the weighting factors shown on the right-hand side of Eq. (15) are simply results of QED. Next, we use the data [3,4] for $F_2^{D(3)}$ which are available at present to perform a log-log plot for the integrand of the expression in Eq. (15) as a function of x_P for different values of β and Q^2 . This is shown in Fig. 6(a) and Fig. 6(c). Since the absolute values of this quantity depend very much, but the slope of the curves very little, on β , we carry out the integration as follows: We first fit every set of the data separately. Having obtained the slopes and the intersection points, we use the obtained fits to perform the integration over β . The results are shown in the

$$\log\left(\frac{1}{x_P} \frac{d^2 \sigma^D}{dQ^2 dx_P}\right) \text{ versus } \log(x_P)$$

plots of Fig. 6(b). These results show that the Q^2 dependence of the slopes is practically negligible, and that the slope is approximately -1.95 for all values of Q^2 .

Furthermore, in order to see whether the quantity introduced in Eq. (15) is indeed useful, and in order to perform a decisive test of the Q^2 independence of the slope in the power-law behavior of the above-mentioned size distributions, we now compare the results in deep-inelastic scattering [3,4] with those obtained in photoproduction [21], where LRG events have also been observed. This means that, as in diffractive deep-inelastic scattering, we again associate the observed effects with colorless objects which are interpreted as a system of interacting soft gluons originating from the proton. In order to find out whether it is the same kind of gluon clusters as in deep-inelastic scattering and whether they “look” very much different when we probe them with real ($Q^2=0$) photons, we replot the existing $d\sigma/dM_x^2$ data [21] for photoproduction experiments performed at different total energies, and note the kinematical relationship between M_x^2 , W^2 , and x_P (for $Q^2 \ll M^2$ and $|t| \ll M_x^2$):

$$x_P \approx \frac{M_x^2}{W^2}. \quad (16)$$

The result of the corresponding

$$\log\left(\frac{1}{x_P} \frac{d\sigma}{dx_P}\right) \text{ versus } \log(x_P)$$

plot is shown in Fig. 7. The slope obtained from a least-squares fit to the existing data [21] is -1.98 ± 0.07 .

The results obtained in diffractive deep-inelastic electron-proton scattering and that for diffractive photoproduction strongly suggest the following: The formation processes of c_0^* 's in the proton are due to self-organized criticality, and thus the spatial distributions of such clusters—represented by the x_P distribution—obey power laws. The exponents of such power laws are independent of Q^2 . Since $1/Q^2$ can be interpreted as a measure for the transverse size of the incident virtual photon, the observed Q^2 independence of the exponents can be considered as further evidence for SOC—in the sense that the self-organized gluon-cluster formation processes take place independent of the photon which is “sent in” to detect the clusters.

Having these results and the close relationship between real photons and hadrons in mind, we are immediately led to the following questions: What shall we see when we replace the (virtual or real) photon by a hadron—a proton or an antiproton? (See in this connection Fig. 8, for the notation and the kinematical relations for the description of such scattering processes.) Should we not see similar behaviors, if SOC in gluon systems is indeed the reason for the occurrence of c_0^* 's which can be probed experimentally in inelastic diffractive scattering processes? To answer these questions, we took a closer look at the available single diffractive proton-proton and proton-antiproton scattering data [5,6], and in order to make quantitative comparisons, we plot the quantities which correspond to those shown in Fig. 6(b) and Fig. 7. These plots are shown in Fig. 9(a) and Fig. 9(b). In Fig. 9(a), we see the double differential cross section $(1/x_P)d^2\sigma/(dt dx_P)$ at four different t values. In Fig. 9(b), we see the integrated differential cross section $(1/x_P)d\sigma/dx_P$. Note that here

$$x_P \approx M_x^2/s, \quad (17)$$

where \sqrt{s} is the total c.m.s. energy of the colliding proton-proton or antiproton-proton system. Here, the integrations of the double differential cross section over t are in the ranges in which the corresponding experiments have been performed. (The extremely weak energy dependence has been ignored in the integration.) The dashed lines in all the plots in Figs. 9(a) and 9(b) stand for the slope -1.97 which is the average of the slope obtained from the plots shown in Figs. 6(b) and 7. This means that the result shows exactly what we expect to see: The fingerprints of SOC can be clearly seen also in proton- and antiproton-induced inelastic diffractive scattering processes, showing that such characteristic features are indeed universal and robust.

We are thus led to the following conclusions. Color-singlet gluon clusters (c_0^*) can be formed in hadrons as a consequence of SOC in systems of interacting soft gluons. In other words, “the colorless objects” which dominate the inelastic diffractive scattering processes are BTW avalanches (BTW clusters). Such c_0^* 's are in general distributed partly inside and partly outside the confinement region of the “mother hadron.” Since the interactions between the c_0^* 's and other color-singlet objects (including the target proton)

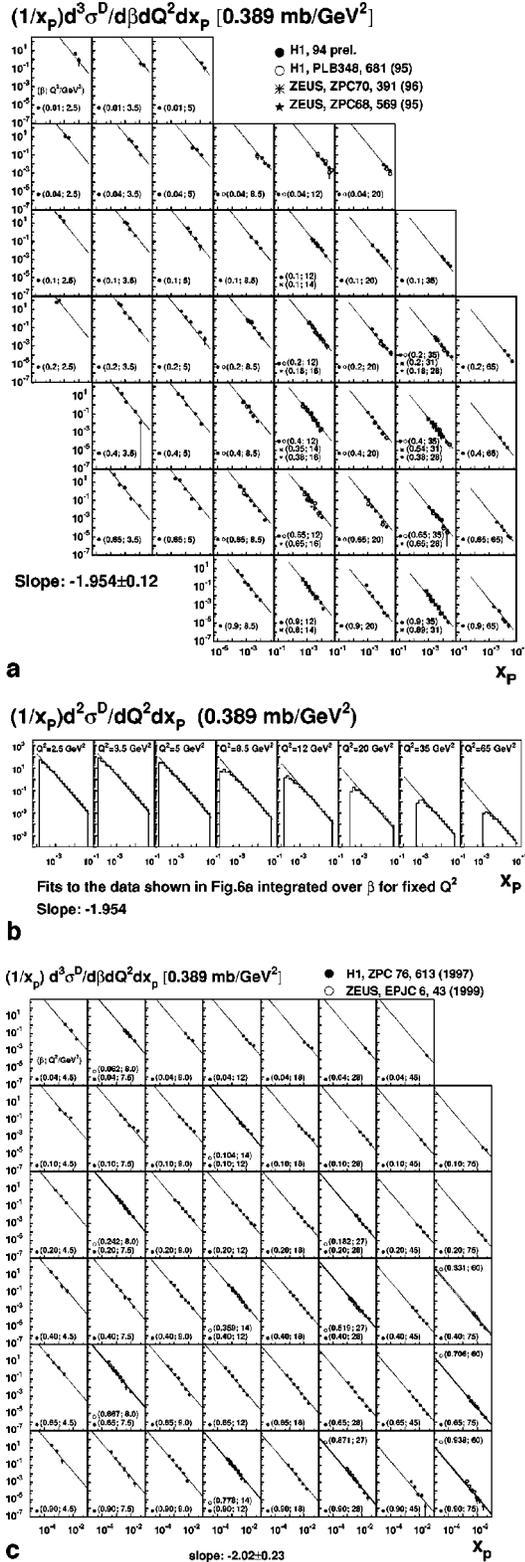


FIG. 6. (a) $(1/x_p)d^3\sigma^D/d\beta dQ^2 dx_p$ is plotted as a function of x_p in different bins of β and Q^2 . The data are taken from Ref. [3]. [The factor 0.389 mb is due to $(1 \text{ GeV})^{-2}=0.389 \text{ mb}$.] (b) $(1/x_p)d^2\sigma^D/dQ^2 dx_p$ is plotted as a function of x_p in different bins of Q^2 . The data are taken from Ref. [3]. (c) Same as (a) but with data taken from Ref. [4]. [The factor 0.389 mb is due to $(1 \text{ GeV})^{-2}=0.389 \text{ mb}$.]

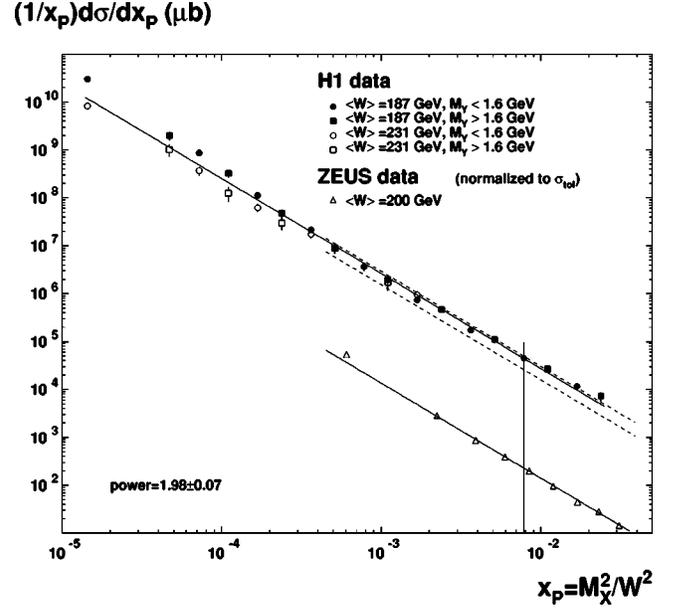


FIG. 7. $(1/x_p)d\sigma/dx_p$ for photoproduction $\gamma+p \rightarrow X+p$ is plotted as a function of x_p . The data are taken from Ref. [21]. Note that the data in the second paper are given in terms of relative cross sections. Note also that the slopes of the straight lines are the same. The two dashed lines indicate the lower and the upper limits of the results obtained by multiplying the lower solid line by $\sigma_{\text{tot}} = 154 \pm 16(\text{stat.}) \pm 32(\text{syst.}) \mu\text{b}$. This value is taken from the third paper in Ref. [21].

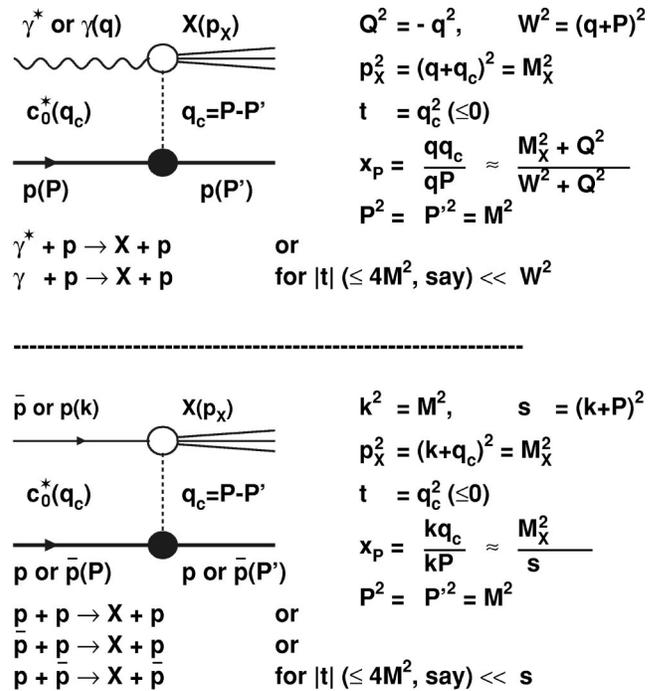


FIG. 8. Diagrams for different single diffractive reactions, together with the definitions of the relevant kinematic variables.

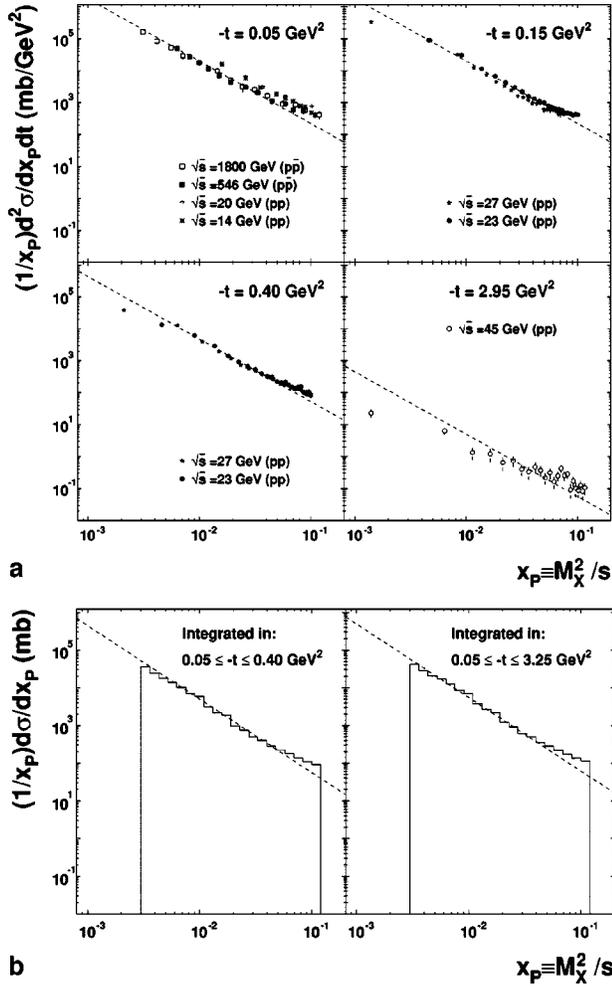


FIG. 9. (a) $(1/x_p)d^2\sigma/dx_p dt$ for single diffractive $p+p \rightarrow p+X$ and $p+\bar{p} \rightarrow p+X$ reactions is plotted as a function of x_p at different values of t and \sqrt{s} . The data are taken from Refs. [5,6]. (b) The integrated (with respect to two different $|t|$ ranges) differential cross section $(1/x_p)d\sigma/dx_p$ for single diffractive $p+p \rightarrow p+X$ and $p+\bar{p} \rightarrow p+X$ reactions is plotted as a function of x_p .

should be of van der Waals type, it is expected that such an object can be readily driven out of the above-mentioned confinement region by the beam particle in geometrically more peripheral collisions. This has been checked by examining inelastic single diffractive scattering processes at high energies in which virtual photon, real photon, proton, and antiproton are used as beam particles. The result of this systematic check shows that the universal distributions of such c_0^* 's can be directly extracted from the data. In particular, given the fact that x_p is the energy fraction carried by the struck c_0^* 's and the fact that the x_p distributions are universal, it is tempting to regard such x_p distributions as the ‘‘parton distributions’’ for diffractive scattering processes. Having seen this, it is also tempting to ask the following: Can we make use of such ‘‘parton distributions’’ to describe and/or to predict the measurable cross sections in inelastic diffractive scattering processes? This and other related questions will be discussed in the following sections.

VIII. DIFFRACTIVE SCATTERING IN HIGH-ENERGY COLLISIONS AND DIFFRACTION IN OPTICS

It might sound strange, but it is true that physicists working in this field of physics often do not agree with one another on the question as to what is diffractive scattering in high-energy collisions. In this paper, we have, until now, simply adopted the currently [9] popular definition of ‘‘inelastic diffractive scattering processes.’’ That is, when we talked about ‘‘inelastic diffractive scattering’’ we were always referring to processes in which ‘‘colorless objects’’ are ‘‘exchanged.’’ In other words, until now, the following question has *not* been asked: Are the above-mentioned ‘‘inelastic diffractive scattering processes’’ indeed comparable with *diffraction* in optics, in the sense that the beam particles should be considered as waves, and the target-proton together with the associated (in whatever manner) colorless objects can indeed be viewed as a ‘‘scattering screen’’?

This question will be discussed in the present and the subsequent sections, together with the existing data [5,22] for the double differential cross section $d^2\sigma/dt d(M_x^2/s)$ for proton-proton and antiproton-proton collisions (where t is the four-momentum-transfer squared, M_x is the missing mass, and \sqrt{s} is the total c.m.s. energy). The purpose of this investigation is to find out the following: ‘‘Can the observed t dependence and the (M_x^2/s) dependence of $d^2\sigma/dt d(M_x^2/s)$ in the given kinematic range ($0.2 \text{ GeV}^2 \leq |t| \leq 3.25 \text{ GeV}^2$, $16 \text{ GeV} \leq \sqrt{s} \leq 630 \text{ GeV}$, and $M_x^2/s \leq 0.1$) be understood in terms of the well-known concept of diffraction in optics? The answer to this question is of particular interest for several reasons.

(a) High-energy proton-proton and proton-antiproton scattering at small momentum transfer has played, and is still playing, a very special role in understanding diffraction and/or diffractive dissociation in lepton-, photon-, and hadron-induced reactions [2–6,8,9,21–24]. Many experiments have been performed at various incident energies for elastic and inelastic diffractive scattering processes. It is known that the double differential cross section $d^2\sigma/dt d(M_x^2/s)$ is a quantity which can yield much information on the reaction mechanism(s) and/or on the structure of the participating colliding objects. In the past, the t , M_x , and s dependence of the differential cross sections for inelastic diffractive scattering processes has been presented in different forms, where a number of interesting features have been observed [5,22,23]. For example, it is seen that the t dependence of $d^2\sigma/dt dM_x^2$ at fixed s depends very much on M_x ; the M_x^2 dependence of $d^2\sigma/dt dM_x^2$ at fixed t depends on s . But when $d^2\sigma/dt d(M_x^2/s)$ is plotted as function of M_x^2/s at given t values (in the range $0.2 \text{ GeV}^2 \leq |t| \leq 3.25 \text{ GeV}^2$) they are approximately independent of s . What do these observed striking features tell us? The first precision measurement of this quantity was published more than 20 years ago [5]. Can this as well as the more recent $d^2\sigma/dt d(M_x^2/s)$ data [22] be understood theoretically?

(b) The idea of using optical and/or geometrical analogies to describe high-energy hadron-nucleus and hadron-hadron collisions at small scattering angles was discussed by many

authors [24,23] many years ago. It is shown in particular that this approach is very useful and successful in describing elastic scattering. However, it seems that, until now, no attempt has been made to describe the data [5,22] by performing quantitative calculations for $d^2\sigma/dtd(M_x^2/s)$ by using optical geometrical analogies. It seems worthwhile to make such an attempt. This is because, it has been pointed out [12] very recently, that the above-mentioned analogy can be made to understand the observed t dependence in $d\sigma/dt$.

(c) Inelastic diffractive pp and $\bar{p}p$ scattering belongs to those soft processes which have also been extensively discussed in the well-known Regge-pole approach [8,9,23]. The basic idea of this approach is that colorless objects in form of Regge trajectories (Pomerons, Reggeons, etc.) are exchanged during the collision, and such trajectories are responsible for the dynamics of the scattering processes. In this approach, it is the t dependence of the Regge trajectories, the t dependence of the corresponding Regge residue functions, the properties of the coupling of the contributing trajectories (e.g., triple Pomeron or Pomeron-Reggeon-Pomeron coupling), and the number of contributing Regge trajectories which determine the experimentally observed t and M_x dependence of $d^2\sigma/dtd(M_x^2/s)$. A number of Regge-pole models [9,8] have been proposed, and there exist good fits [9,8] to the data. What remains to be understood in this approach is the dynamical origin of the Regge trajectories, on the one hand, and the physical meaning of the unknown functions (for example the t dependence of any one of the Regge-residue functions), on the other. It has been pointed out [20,12], that there may be an overlap between the ‘‘partons in Pomeron and Reggeons’’ picture and the SOC picture [12], and that one way to study the possible relationship between the two approaches is to take a closer look at the double differential cross section $d^2\sigma/dtd(M_x^2/s)$.

IX. OPTICAL DIFFRACTION OFF DYNAMICAL COMPLEX SYSTEMS

Let us begin our discussion of the above-mentioned questions by recalling that the concept of ‘‘diffraction’’ or ‘‘diffractive scattering’’ has its origin in optics, and optics is part of electrodynamics, which is not only the *classical limit*, but also *the basis* of quantum electrodynamics (QED). Here, it is useful to recall in particular the following: Optical diffraction is associated with a departure from geometrical optics caused by the finite wavelength of light. Fraunhofer diffraction can be observed by placing a scatterer (which can in general be a scattering screen with more than one aperture or a system of scattering objects) in the path of propagation of light (the wavelength of which is less than the linear dimension of the scatterer) where not only the light source, but also the detecting device, is very far away from the scatterer. The parallel incident light rays can be considered as plane waves (characterized by a set of constants $\vec{k}, \omega \equiv |\vec{k}|$, and u , say, which denote the wave vector, the frequency, and the amplitude of a component of the electromagnetic field, respectively, in the laboratory frame). After the scattering, the scattered field can be written in accordance with Huygens’ principle as

$$u_P = \frac{e^{i|\vec{k}'|R}}{R} f(\vec{k}, \vec{k}'). \quad (18)$$

Here, u_P stands for a component of the field originating from the scatterer, \vec{k}' is the wave vector of the scattered light in the direction of observation, $|\vec{k}'| \equiv \omega'$ is the corresponding frequency, R is the distance between the scatterer and the observation point P , and $f(\vec{k}, \vec{k}')$ is the (unnormalized) scattering amplitude which describes the change of the wave vector in the scattering process. By choosing a coordinate system in which the z axis coincides with the incident wave vector \vec{k} , the scattering amplitude can be expressed as follows [25,24,23]:

$$f(\vec{q}) = \frac{1}{(2\pi)^2} \int_{\Sigma} \int d^2\vec{b} \alpha(\vec{b}) e^{-i\vec{q}\cdot\vec{b}}. \quad (19)$$

Here, $\vec{q} \equiv \vec{k}' - \vec{k}$ determines the change in wave vector due to diffraction, \vec{b} is the impact parameter which indicates the position of an infinitesimal surface element on the wavefront ‘‘immediately behind the scatterer’’ where the incident wave would reach in the limit of geometrical optics, and $\alpha(\vec{b})$ is the corresponding amplitude (associated with the boundary conditions which the scattered field should satisfy) in the two-dimensional impact-parameter space (which is here the xy plane), and the integration extends over the region Σ in which $\alpha(\vec{b})$ is different from zero. In those cases in which the scatterer is symmetric with respect to the scattering axis (here the z axis), Eq. (19) can be expressed, by using an integral representation for J_0 , as

$$f(q) = \frac{1}{2\pi} \int_0^\infty b db \alpha(b) J_0(qb), \quad (20)$$

where q and b are the magnitudes of \vec{q} and \vec{b} , respectively.

The following should be mentioned in connection with Eqs. (19) and (20): Many of the well-known phenomena related to Fraunhofer diffraction have been deduced [25] from these equations under the additional condition (which is directly related to the boundary conditions imposed on the scattered field) $|\vec{k}'| = |\vec{k}| = \omega' = \omega$; that is, \vec{k}' differs from \vec{k} only in direction. In other words, the outgoing light wave has exactly the same frequency and exactly the same magnitude of wave vector as those for the incoming wave. (This means that, quantum mechanically speaking, the outgoing photons are also on-shell photons, the energies of which are the same as the incoming ones.) In such cases, it is possible to envisage that \vec{q} is approximately perpendicular to \vec{k} and to \vec{k}' ; that is, \vec{q} is approximately perpendicular to the chosen z axis and thus in the above-mentioned xy plane (that is $\vec{q} \approx \vec{q}_\perp$). While the scattering angle distribution in such processes (which are considered as the characteristic features of *elastic* diffractive scattering) plays a significant role in understanding the observed diffraction phenomena, it is of considerable importance to note the following.

(A) Equations (19) and (20) can be used to describe diffractive scattering with or without this additional condition, provided that the difference of \vec{k}' and \vec{k} in the longitudinal direction (i.e., in the direction of \vec{k}) is small compared to $q_{\perp} \equiv |\vec{q}_{\perp}|$ so that \vec{q}_{\perp} can be approximated by \vec{q} . In fact, Eqs. (19) and (20) are strictly valid when \vec{q} is a vector in the above-mentioned xy plane, that is, when we write \vec{q}_{\perp} instead of \vec{q} . Now, since Eqs. (19) and (20) in such a form (that is, when the replacement $\vec{q} \rightarrow \vec{q}_{\perp}$ is made) are valid *without* the condition that \vec{q} should approximately be equal to \vec{q}_{\perp} and in particular without the additional condition $|\vec{k}'| = |\vec{k}| = \omega' = \omega$, it is clear that they are also valid for *inelastic* scattering processes. In other words, Eqs. (19) and (20) can also be used to describe *inelastic* diffractive scattering (that is, processes in which $\omega' \neq \omega$, $|\vec{k}'| \neq |\vec{k}|$) provided that the following replacements are made. In Eq. (19), $\vec{q} \rightarrow \vec{q}_{\perp}$, $f(\vec{q}) \rightarrow f_{\text{inel}}(\vec{q}_{\perp})$, $\alpha(\vec{b}) \rightarrow \alpha_{\text{inel}}(\vec{b})$, and in Eq. (20), $q \rightarrow q_{\perp}$, $f(q) \rightarrow f_{\text{inel}}(q_{\perp})$, $\alpha(b) \rightarrow \alpha_{\text{inel}}(b)$. Hereafter, we shall call Eqs. (19) and (20) with these replacements Eqs. (19') and (20'), respectively. We note that in order to specify the dependence of f_{inel} on ω' and k'_{\parallel} (that is on $\omega' - \omega$ and $k'_{\parallel} - k_{\parallel}$), further information on energy-momentum transfer in such scattering processes is needed. This point will be discussed in more detail in Sec. X.

(B) In scattering processes at large momentum transfer where the magnitude of \vec{q}_{\perp} is large ($|\vec{q}_{\perp}|^2 \gg 0.05 \text{ GeV}^2$, say), it is less probable to find diffractive scattering events in which the additional condition $|\vec{k}'| = |\vec{k}|$ and $\omega' = \omega$ can be satisfied. This means that it is expected that most of the diffraction phenomena observed in such processes are associated with inelastic diffractive scattering.

(C) A change in angle but no change in magnitude of wave vectors or frequencies is likely to occur in processes in which neither absorption nor emission of light takes place. Hence, it is not difficult to imagine that the above-mentioned condition can be readily satisfied in cases where the scattering systems are time-independent macroscopic apertures or objects. But in this connection, we are also forced to the following question: How large is the chance for an incident wave *not* to change the magnitude of its wave vector in processes in which the scatterers are *open, dynamical, complex systems*, where energy and momentum exchanges take place at anytime and everywhere?

The picture for inelastic diffractive scattering has two basic ingredients.

First, having the well-known phenomena associated with Fraunhofer diffraction and the properties of de Broglie matter waves in mind, the beam particles (γ^* , γ , \bar{p} , or p shown in Fig. 8) in these scattering processes are considered as high-frequency waves passing through a medium. Since, in general, energy and momentum transfers take place during the passage through the medium, the wave vector of the outgoing wave differs, in general, from the incoming one, not only in direction, but also in magnitude. For the same reason, the frequency and longitudinal components of the wave vector of the outgoing wave (that is, the energy and/or the in-

variant mass, as well as the longitudinal momentum of the outgoing particles) can be different from their incoming counterparts.

Second, according to the results obtained in Secs. I–VII of this paper, the medium is a system of color-singlet gluon clusters (c_0^* 's) which are in general partly inside and partly outside the proton—in form of a “cluster cloud.” Since the average binding energy between such color-singlet aggregates are of van der Waals type [26], and thus it is negligibly small compared with the corresponding binding energy between colored objects, we expect to see that, even at relatively small values of momentum transfer ($|t| < 1 \text{ GeV}^2$, say), the struck c_0^* can unify with (be absorbed by) the beam particle and “be carried away” by the latter, similar to the process of “knocking out nucleons” from nuclear targets in high-energy hadron-nucleus collisions. It should, however, be emphasized that, in contrast to the nucleons in nucleus, the c_0^* 's which can exist inside or outside the confinement region of the proton are *not* hadron like (see Secs. III–VI for more details). They are BTW avalanches which have neither a typical size nor a typical lifetime, nor a given static structure. Their size and lifetime distributions obey simple power laws as a consequence of SOC. This means that in the diffraction processes discussed here, the size of the scatterer(s) and thus the size of the carried-away c_0^* are in general different in every scattering event. It should also be emphasized that these characteristic features of the scatterer are consequences of the basic properties of the gluons.

X. CAN SUCH SCATTERING SYSTEMS BE MODELED QUANTITATIVELY?

To model the proposed picture quantitatively, it is convenient to consider the scattering system in the rest frame of the proton target. Here, we choose a right-handed Cartesian coordinate with its origin O at the center of the target proton and the z axis in the direction of the incident beam. The xy plane in this coordinate system coincides with the two-dimensional impact-parameter space mentioned in connection with Eqs. (19') and (20') [which are respectively Eqs. (19) and (20) after the replacements mentioned in (A) below Eq. (20)], while the yz plane is the scattering plane. We note that since we are dealing with inelastic scattering (where the momentum transfer, including its component in the longitudinal direction, can be large, in accordance with the uncertainty principle), it is possible to envisage that (the c.m.s. of) the incident particle in the beam meets c_0^* 's at one point $B \equiv (0, b, z)$, where the projection of OB along the y axis characterizes the corresponding impact parameter \vec{b} . We recall that such c_0^* 's are avalanches initiated by local perturbations (caused by local gluon interactions associated with absorption or emission of one or more gluons; see Secs. I–VII for details) of SOC states in systems of interacting soft gluons. Since gluons carry color, the interactions which lead to the formation of *color-singlet* gluon clusters (c_0^*) must take place inside the confinement region of the proton. This means that while a considerable part of such c_0^* 's in the cloud can be outside the proton, the location A , where such

an avalanche is initiated, *must* be *inside* the proton. That is, in terms of $OA \equiv r$, $AB \equiv R_A(b)$, and proton's radius r_p , we have $r \leq r_p$ and $[R_A(b)]^2 = b^2 + z^2 + r^2 - 2(b^2 + z^2)^{1/2}r \cos \angle BOA$. For a given impact parameter b , it is useful to know the distance $R_A(b)$ between B and A , as well as "the average squared distance" $\langle R_A^2(b) \rangle = b^2 + z^2 + a^2$, $a^2 \equiv 3/5r_p^2$, which is obtained by averaging over all allowed locations of A in the confinement region. That is, we can model *the effect of confinement* in cluster formation by picturing that all the avalanches, in particular those which contribute to scattering events characterized by a given b and a given z , are initiated from an "effective initial point" $\langle A_B \rangle$, because only the mean distance between A and B plays a role. (We note that since we are dealing with a complex system with many degrees of freedom, in which B as well as A are randomly chosen points in space, we can compare *the mean distance* between B and A with the mean free path in a gas mixture of two kinds of gas molecules—"species B " and "species A ," say, where those of the latter kind are confined inside a subspace called "region p ." For a *given mean distance* and a *given point B*, there is in general a set of A 's inside the "region p ," such that their distance to B is *equal* to the given mean value. Hence it is useful to introduce a *representative point* $\langle A_B \rangle$, such that the distance between $\langle A_B \rangle$ and B is equal to the given mean distance.) Furthermore, since an avalanche is a dynamical object, it may propagate within its lifetime in any one of the 4π directions away from $\langle A_B \rangle$. (Note that avalanches of the same size may have different lifetimes and different structures, as well as different shapes. The location of an avalanche in space-time is referred to its center of mass.) Having seen how SOC and confinement can be implemented in describing the properties and the dynamics of the c_0^* 's, which are nothing else but BTW avalanches in systems of interacting soft gluons, let us now go one step further, and discuss how these results can be used to obtain the amplitudes in impact-parameter space that leads, via Eq. (20), to the scattering amplitudes.

In contrast to the usual cases, where the scatterer in the optical geometrical picture of a diffractive scattering process is an aperture or an object with a given static structure, the scatterer in the proposed picture is an open, dynamical, complex system of c_0^* 's. This implies, in particular, that the object(s), which the beam particle hits, has (have) neither a typical size nor a typical lifetime, nor a given static structure.

With these in mind, let us now come back to our discussion of the double differential cross section $d^2\sigma/dt d(M_x^2/s)$. Here, we need to determine the corresponding amplitude $\alpha_{\text{inel}}(b)$ in Eq. (20') [see the discussion in (A) below Eq. (20)]. What we wish to do now is to focus our attention on those scattered matter waves whose de Broglie wavelengths are determined by the energy momentum of the scattered object, whose invariant mass is M_x . For this purpose, we characterize the corresponding $\alpha_{\text{inel}}(b)$ by considering it as a function of M_x or M_x^2/s , or x_p . We recall in this connection that, for inelastic diffractive scattering processes in hadron-hadron collisions, the quantity M_x^2/s is approximately equal to x_p , which is the momentum fraction carried by the struck c_0^* 's with respect to the incident beam (see Fig. 8 for more

details; note, however, that q_c , k , and p_x in Fig. 8 correspond, respectively, to q , k , and k' in the discussions here). Hence, we shall write hereafter $\alpha(b|M_x^2/s)$ or $\alpha(b|x_p)$ instead of the general expression $\alpha_{\text{inel}}(b)$. This, together with Eq. (20'), leads to the corresponding scattering amplitude $f(q_\perp|x_p)$ and thus to the corresponding double differential cross section $d^2\sigma/dt dx_p$, in terms of the variables $|t| \approx |\vec{q}_\perp|^2$ and $x_p \approx M_x^2/s$ in the kinematical region: $|t| \ll M_x^2 \ll s$.

XI. ROLE PLAYED BY THE SPACE-TIME PROPERTIES OF THE GLUON CLUSTERS

For the determination of $\alpha(b|x_p)$, it is of considerable importance to recall the following space-time properties of the c_0^* 's which are BTW avalanches due to SOC.

(i) SOC dictates that there are BTW avalanches of all sizes (which we denote by different S values), and that the probability amplitude of finding an avalanche of size S can be obtained from the size distribution $D_S(S) = S^{-\mu}$ where the experimental results presented in Secs. I–VII show $\mu \approx 2$. This means that $D_S(S)$ contributes a factor S^{-1} and thus a factor x_p^{-1} to the scattering amplitude $\alpha(b|x_p)$. Here, as well as in (ii), we take into account (see Secs. IV and V for details) that the size S of a c_0^* is directly proportional to the total amount of the energy the c_0^* carries; the amount of energy is $x_p P^0$, where P^0 is the total energy of the proton, and x_p is the energy fraction carried by the c_0^* .

(ii) QCD implies [26] that the interactions between two arbitrarily chosen colored constituents of c_0^* are stronger than those between two c_0^* 's, because the latter should be interactions of Van der Waals type. This means that the struck c_0^* can unify with the beam particle (maybe by absorbing each other), and viewed from any Lorentz frame in which the beam particle has a larger momentum than the c_0^* , the latter is "carried away" by the beam particle. Geometrically, the chance for the beam particle to hit an c_0^* of size S (on the plane perpendicular to the incident axis) is proportional to the area that can be struck by the (c.m.s.) of the beam particle. The area is the $2/3$ power of the volume S , $S^{2/3}$, and thus it is proportional to $x_p^{2/3}$.

(iii) Based on the above-mentioned picture in which the c_0^* 's propagate isotropically from $\langle A_B \rangle$, the relative number densities at different b values can be readily evaluated. Since for a given b the distance in space between $\langle A_B \rangle$ and $B \equiv (0, b, z)$ is simply $(b^2 + z^2 + a^2)^{1/2}$, the number of c_0^* 's which pass a unit area on the shell of radius $(b^2 + z^2 + a^2)^{1/2}$ centered at $\langle A_B \rangle$ is proportional to $(b^2 + z^2 + a^2)^{-1}$, provided that (because of causality) the lifetimes (T 's) of these c_0^* 's are not shorter than $\tau_{\text{min}}(b)$. The latter is the time interval for a c_0^* to travel from $\langle A_B \rangle$ to B . This means that because of the space-time properties of such c_0^* 's, it is of considerable importance to note that, first, only c_0^* 's having lifetimes $T \geq \tau_{\text{min}}(b)$ can contribute to such a collision event. Second, during the propagation from $\langle A_B \rangle$ to B , the motion of such a c_0^* has to be considered as Brownian. In fact, the continual, and more or less random, impacts received from

the neighboring objects on its path lead us to the well-known [27] result that the time elapsed is proportional to the mean-square displacement. That is, $\tau_{\min}(b) \propto b^2 + z^2 + a^2$. Furthermore, we recall that c_0^* 's are due to SOC, and thus the chance for a c_0^* of lifetime T to exist is $D_T(T) \propto T^{-\nu}$ where the experimental value (see Secs. I–VII) for ν is $\nu \approx 2$. Hence, by integrating T^{-2} over T from $\tau_{\min}(b)$ to infinity, we obtain the fraction associated with all those whose lifetimes satisfy $T \geq \tau_{\min}(b)$: This fraction is $\tau_{\min}(b)^{-1}$ and thus a constant times $(b^2 + z^2 + a^2)^{-1}$.

The amplitude $\alpha(b|x_p)$ can now be obtained from the probability amplitude for c_0^* -creation mentioned in (i), by taking the weighting factors mentioned in (ii) and (iii) into account, and by integrating [28] over z . The result is

$$\alpha(b|x_p) = \text{const} \times x_p^{-1/3} (b^2 + a^2)^{-3/2}. \quad (21)$$

By inserting this probability amplitude in impact-parameter space, for the beam particle to encounter a c_0^* , which carries a fraction x_p of the proton's total energy, in Eq. (20') [which is Eq. (20) with the following replacements: $q \rightarrow q_\perp$, $f(q) \rightarrow f_{\text{inel}}(q|x_p)$ and $\alpha(b) \rightarrow \alpha_{\text{inel}}(b|x_p)$] we obtain the corresponding probability amplitude $f(q|x_p)$ in momentum space:

$$f(q_\perp|x_p) = \text{const} \times \int_0^\infty b db x_p^{-1/3} (b^2 + a^2)^{-3/2} J_0(q_\perp b), \quad (22)$$

where $q_\perp = |\vec{q}_\perp| \approx \sqrt{|t|}$ (in the small x_p region, $x_p < 0.1$, say) is the corresponding momentum transfer. The integration can be carried out analytically [29], and the result is

$$f(q_\perp|x_p) = \text{const} \times x_p^{-1/3} \exp(-aq_\perp). \quad (23)$$

Hence, the corresponding double differential cross section $d^2\sigma/dt dx_p$ can approximately be written as

$$\frac{1}{\pi} \frac{d^2\sigma}{dt dx_p} = N x_p^{-2/3} \exp(-2a\sqrt{|t|}), \quad (24)$$

where N is an unknown normalization constant. Because of the kinematical relationship $x_p \approx M_x^2/s$ for single diffractive scattering in proton-proton and proton-antiproton collisions (see Fig. 8 for more details), this can be, and should be, compared with the measured double differential cross sections $d^2\sigma/dtd(M_x^2/s)$ at different t and s values and for different missing masses M_x in the region $M_x^2/s \ll 1$ where q_\perp is approximately $\sqrt{|t|}$. The comparison is shown in Fig. 10. Here, we made use of the fact that $a^2 \equiv 3/5 r_p^2$, where r_p is the proton radius, and calculated a by setting r_p^2 to be the well-known [30] mean-square proton charge radius, the value of which is $r_p^2 = (0.81 \text{ fm})^2$. The result we obtained is $a = 3.2 \text{ GeV}^{-1}$. The unknown normalization constant is determined by inserting this calculated value for a in Eq. (24), and by comparing the right-hand side of this equation with the $d^2\sigma/dtd(M_x^2/s)$ data taken at $|t| = 0.2 \text{ GeV}^2$. The value is $N = 31.1 \text{ mb GeV}^{-2}$. All the curves shown in Fig. 10 are obtained by inserting these values for a and N in Eq. (24).

While the quality of the obtained result, namely, the expression given on the right-hand side of Eq. (24) together with the above-mentioned values for a and N , can be readily judged by comparing it with the data or by counting the unknown parameters, or both, it seems worthwhile to recall the following: The two basic ingredients of the proposed picture which have been used to derive this simple analytical expression are, first, the well-known optical analogy and, second, the properties of the dynamical scattering system. The latter is what we have learned through the data analysis presented in Secs. I–VII.

Based on the theoretical arguments and experimental indications for the observation (see Ref. [12] and Secs. I–VII of this paper for details) that the characteristic features of inelastic diffractive scattering processes are approximately independent of the incident energy and independent of the quantum numbers of the beam particles, the following results are expected: The explicit formula for the double differential cross section as shown in Eq. (24) should also be valid for the reactions $\gamma p \rightarrow Xp$ and $\gamma^* p \rightarrow Xp$. While the normalization constant N (which should in particular depend on the geometry of the beam particle) is expected to be different for different reactions, everything else—especially the ‘‘slope’’ as well as the power of x_p —should be exactly the same as in pp and $p\bar{p}$ collisions. In this sense, Eq. (24) with $a^2 = 3/5 r_p^2$ (r_p is the proton radius) is our prediction for $\gamma p \rightarrow Xp$ and $\gamma^* p \rightarrow Xp$ which can be measured at HERA.

Furthermore, in order to obtain the integrated differential cross section $d\sigma/dt$, which has also been measured for different reactions at different incident energies, we only need to sum and integrate over x_p in the given kinematic range ($x_p < 0.1$, say). The result is

$$\frac{d\sigma}{dt}(t) = C \exp(-2a\sqrt{|t|}), \quad (25)$$

where C is an unknown normalization constant. While this observation has already been briefly discussed in the previous Letter [12], we now show the result of a further test of its universality: In Fig. 11, we plot

$$-\frac{1}{2\sqrt{|t|}} \log \left[\frac{1}{C} \frac{d\sigma}{dt}(t) \right] \text{ vs } t \quad (26)$$

for different reactions at different incident energies in the range $0.2 \text{ GeV}^2 \leq |t| \leq 4 \text{ GeV}^2$. Here we see in particular that measurements of $d\sigma/dt$ for $\gamma^* p$ and γp reactions at larger $|t|$ values would be very useful.

XII. CONCLUDING REMARKS

Based on the characteristic properties of the gluons—in particular the local gluon-gluon coupling prescribed by the QCD Lagrangian, the confinement, and the nonconservation of gluon numbers, we suggest that a system of interacting soft gluons should be considered as an open dynamical complex system which is in general far away from equilibrium. Taken together with the observations made by Bak, Tang, and Wiesenfeld [13,14], we are led to the conclusion that

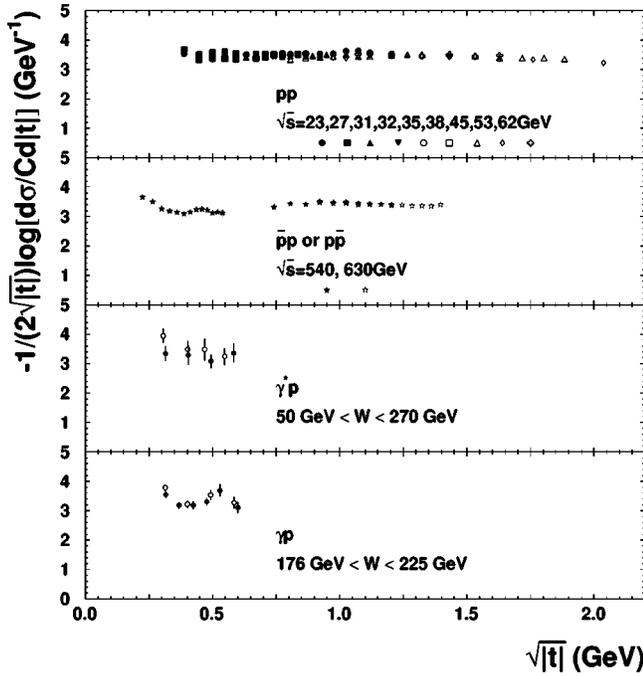


FIG. 11. The quantity $[-1/(2\sqrt{|t|})]\log[(1/C)d\sigma/dt]$ is plotted versus $\sqrt{|t|}$ for different single diffractive reactions in the range $0.2 \text{ GeV}^2 \leq |t| \leq 4 \text{ GeV}^2$. The data are taken from Refs. [5,22,31]. Here, C , the normalization constant, is first determined by performing a two-parameter fit of the corresponding $d\sigma/dt$ data to Eq. (25).

ness of SOC. Hence, the following picture emerges: For the beam particle (which may be a virtual photon, a real photon, a proton, or an antiproton; see Fig. 8 for more details) in an inelastic diffractive scattering process off protons (one may wish to view this from a “fast moving frame” such as the c.m.s. frame), the target proton appears as a cloud of c_0^* 's which exist inside and outside the confinement region of the proton. The size distribution $D_S(S)$ and the lifetime distribution $D_T(T)$ can be expressed as $S^{-\mu}$ and $T^{-\nu}$, respectively,

where the empirical values for μ and ν are $\mu \approx \nu \approx 2$, independent of the incident energy and independent of the quantum numbers of the beam particles.

What do we learn from this? Is this knowledge helpful in understanding hadronic structure and/or hadronic reactions in particle physics? In particular, can this knowledge be used to do *quantitative* calculations—especially those where the results of which could *not* be achieved otherwise?

In order to demonstrate how the obtained knowledge can be used to relate hadron structure and hadronic reactions in general, and to perform quantitative calculations in particular, we discuss the following question—a question which has been with the high-energy physics community for many years: Can the measured double differential cross section $d^2\sigma/dt d(M_x^2/s)$ for inelastic diffractive scattering in proton-proton and in antiproton-proton collisions, in the kinematical region given by $0.2 \text{ GeV}^2 \leq |t| \leq 3.25 \text{ GeV}^2$, $16 \text{ GeV} \leq \sqrt{s} \leq 630 \text{ GeV}$, and $M_x^2/s \leq 0.1$, be understood in terms of optical geometrical concepts?

The answer to this question is yes, and the details are presented in Secs. IX–XI where the following is explicitly shown: The characteristic features of the existing $d^2\sigma/dt d(M_x^2/s)$ data are very much the same as those in optical diffraction, provided that the high-energy beams are considered as high-frequency waves, and the scatterer is a system of color-singlet gluon clusters (c_0^*) described in Secs. I–VII of this paper. Further measurements of double differential cross sections, especially in γ^*p and γp reactions, will be helpful in testing the ideas presented here.

ACKNOWLEDGMENTS

We thank P. Bak, X. Cai, D.H.E. Gross, C.S. Lam, Z. Liang, K.D. Schotte, C.B. Yang, E. Yen, and W. Zhu for helpful discussions, R.C. Hwa, C.S. Lam, and J. Pan for correspondence, and FNK der FU-Berlin for financial support. Y.Z. also thanks Alexander von Humboldt Stiftung for financial support.

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